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RESEARCH

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Hydromagnetic flow of micropolar nanofluids with co-effects of thermal radiation and chemical reaction over an inclined permeable stretching surface

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Abstract

Background Many investigations have been conducted by researchers across the globe to examine the behavior of fluids with respect to the influence of some constituent parameters and novel results have been obtained. However, the combined effect of thermal radiation and chemical reaction on micropolar nanofluid flow over an inclined stretching surface has not been well elucidated. This article, therefore, employed the mathematical model of Buongiorno for hydromagnetic micropolar nanofluids to study the effect of thermal radiation and chemical reaction on such fluids. The model examined the influence of thermophoresis, Brownian motion and the angle of inclination to the stretching surface on the fluid flow. The set of governing equations were transformed into ordinary differential equations using some similarity transformations and then numerically simplified through Chebyshev collocation method on MATHEMATICA software.

Results The graphs thus derived were used to interpret the effect of some physical parameters on the fluid flow. It was observed among other results obtained, that thermal radiation, Brownian motion and thermophoresis enhanced the temperature profile of the flow while the inclination angle and chemical reaction declined the velocity and concentration, respectively.

Conclusions These parameters tested on the various profiles proved observably effective on micropolar nanofluids and should be considered whenever improvement or decrease in the profiles are needed.

Keywords Micropolar nanofluid, Chemical reaction, Thermal radiation, Stretching surface

1 Background

In recent times, investigations on nanofluids and nanotechnology have become interesting and thus greatly attracted the attentions of researchers and experts across the globe. This is owing to its efficiency in handling heat and mass transfer procedures within the fluid. Some of these procedures include polymer processes, films heating and cooling, extraction of crude oil and electronic engineering among others. From the root words "nano" and "fluids", these thermal efficient fluids known as nanofluids are derived from the mixture of some nanometer-sized particles (less than 100 nm) with other choice base fluids such as water, oil and ethylene glycol among others. The nanoparticles basically serve the purpose of enhancing the thermal conductivity of the base fluids.

Micropolar fluids are characterized with the possession of some microconstituents which can rotate within the molecules of the fluid to influence the hydrodynamics

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of the flow. Such liquids have firm arbitrarily directed elements with microstructural components that when immersed in a moderately sticky liquid, possible distortions would not be easily noticed. They flow symmetrically and the exact symmetric solutions are synonymous to the normal Navier–Stokes equation except that it is not always realizable, hence the need to employ the Buongiorno model. This has made micropolar fluids to be of preference in industrial processes for colloidal solutions, paints, polymeric fluids and so on.

Research on boundary layer flow of fluid on flat surfaces was initiated [1]. The convection term in the momentum equation was simplified on the basis of a hypothesis that all convections occur at the same velocity with the moving object. Studies on fluid flow over a continuously moving surface at constant, variable viscosity and velocity showed against the assumption of many researchers that stretching sheets can be extensible and that the production of polymer would only require a means to handle the stretching material [2–4]. The industrial process for the production of some heat-treated materials, metallurgy and aerodynamic extrusion require appropriate cooling fluids that will help to manage the effects of heat on the expected product. The initial research on boundary layer flow over flat surfaces was improved by introducing suction and injection parameters into the flow of the fluid alongside considering the sheet to be stretched with a linear speed [5]. The flow behavior of nanofluids, for example, lubricants, oil with nano-suspensions, paints, blood with corpuscles among others was scrutinized [6]. Some laudable research contributions are reported in [7–14].

The flow of a micropolar fluid due to a porous stretching sheet and heat transfer was examined [15]. Thermal radiation and convective heating on hydromagnetic boundary layer flow of nanofluids over a permeable stretching surface was analyzed [16]. It was observed that copper nanofluids were very good adsorbents of heat and hence can be preferred to other fluids when cooling is required. Recently, Keller–Box simulation for the Buongiorno mathematical model of micropolar nanofluids flow over a nonlinear inclined surface was examined and much attention was drawn to the efficiency of the numerical methods used [17]. Also, a commendable effort was harnessed into the research titled 'hydromagnetic flow of micropolar nanofluids' and a novel result was obtained [18]. Other relevant findings are as well reported [19–22].

A unique method to handle differential equations possessing fractal-fractional characteristics was analyzed [23], and a total mathematical composition of Sutterby fluid flow which are brought about magnetically and immersed in a doubly stratified plate over a stretching surface that is linear was examined [24]. Derivatives of fractal-fractional order for models of HIV/AIDS with Mittag–Leffler kernel was studied [25] and also methods of dual approximation for fractional order false-parabolic differential equations was considered [26]. Mathematical study of the effect of interfacial nano layers and Lorentz force on the flow of nanofluids through orthogonal porous surfaces with injection of SWCNTs was analyzed [27], and magnetized flow of viscous fluid in a permeable path with single wall carbon nanotubes dispersion by using nano-layer approach was examined [28].

Of all these literature, no particular work has been done to check the combined effects of thermal radiation and chemical reaction on hydromagnetic boundary layer flow of micropolar nanofluids over an inclined stretching surface which is the motivation to this present research.

2 Methods

The work is set to examine the hydromagnetic flow of nanofluids with micropolar characteristics over a permeable stretching surface which is inclined at an angle γ, where $'x'$ is the coordinate in the direction of the extending surface and $'a'$ is known as a constant. The velocities considered are $u_w(x) = ax$ and $u_\infty(x) = 0$ at the wall and far away from the wall, respectively, the transverse magnetic field is taken at right angle to the stretching surface on the assumption that the electric and magnetic field effects are negligible because of the small magnetic Reynolds number. The fluid particles which comprise finite size of micropolar particles and nanoparticles are freely distributed within the base fluids where they hit one another and rotate in the fluid fields. The nanoparticle fraction and temperature denoted as C and T take constant values of C_w and T_w with C_∞ and T_∞ as mass fraction and temperature at the wall and far away from the wall, respectively. The flow geometry is represented in the figure below (Fig. 1).

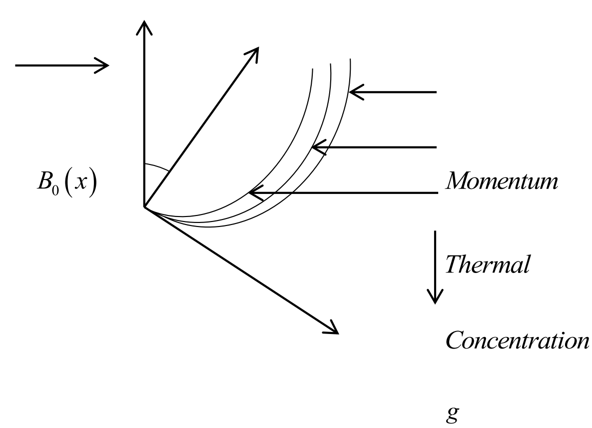

Fig. 1 Geometry of the flow

The governing equations for the flow are represented below:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(\frac{\mu+K_1^*}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\left(\frac{K_1^*}{\rho}\right)\frac{\partial N^*}{\partial y}+g[\beta_t(T-T_\infty)+\beta_c(C-C_\infty)]\cos\gamma-\left(\frac{\sigma B_0^2}{\rho}\right)u-\frac{v^*}{K_1^*}u \tag{2}$$

$$u\frac{\partial N^*}{\partial x}+v\frac{\partial N^*}{\partial y}=\left(\frac{\gamma^*}{j^*\rho}\right)\frac{\partial^2 N^*}{\partial y^2}-\left(\frac{K_1^*}{j^*\rho}\right)\left(2N^*+\frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{Q_0}{\rho c_p}(T-T_\infty) \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\tau\left(D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}\right)-R^*(C-C_\infty) \tag{5}$$

The velocity components along x and y are represented as u and v, Q_0 is the heat absorption or generation coefficient, the thermal diffusivity parameter represented as $\alpha=\frac{k}{(\rho c)_f}$, R^* is denoted as the chemical reaction and the ratio of the effective heat capacity of the nanoparticle to that of the liquid as $\tau=\frac{(\rho c)_p}{(\rho c)_f}$.

The boundary conditions are given below:

$$\begin{aligned}&u=u_w(x)=ax,\quad v=V_w,\quad T=T_w,\\&N^*=-m_0\frac{\partial u}{\partial y},\quad C=C_w\quad \text{at } y=0\\&u\to u_\infty(x)=0,\quad v\to 0,\quad T\to T_\infty,\quad N^*\to 0,\\&C\to C_\infty\quad \text{at } y\to\infty\end{aligned} \tag{6}$$

$$\text{Taking } \psi=\psi(x,y) \text{ as stream function, where } u=\frac{\partial\psi}{\partial y} \text{ and } v=-\frac{\partial\psi}{\partial x} \tag{7}$$

Equation (1) is identically satisfied. The transformations will require the following

$$\eta=y\sqrt{\frac{a}{\upsilon}},\ N^*=ax\sqrt{\frac{a}{\upsilon}}h(\eta),\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty},\ u=axf'(\eta),\ v=-\sqrt{a\upsilon}f(\eta) \tag{8}$$

Resolving the set of Eqs. (2)–(5) by substituting Eq. (8), we would obtain

$$(1+k)f'''+ff''-f'^2-(M+\varphi)f'+kh'+(Gr_x\theta+Gc_x\phi)\cos\gamma=0 \tag{9}$$

$$\left(1+\frac{k}{2}\right)h''+fh'-f'h-k\left(2h+f''\right)=0 \tag{10}$$

$$\frac{(1+R)}{\mathrm{Pr}}\theta''+\lambda\theta+f\theta'+Nb\theta'\phi'+Nt\theta'^2=0 \tag{11}$$

$$\phi''+\frac{Nt}{Nb}\theta''+Lef\phi'-RnLe\phi=0 \tag{12}$$

Some other substitutions involved in Eqs. (9)–(12) were obtained from the dimensionless variables below

$$\begin{aligned}&M=\frac{\sigma B_0^2}{a\rho},\ Le=\frac{\upsilon}{D_B},\ k=\frac{k_1^*}{\mu},\ \mathrm{Pr}=\frac{\upsilon}{\alpha},\\&\lambda=\frac{Q_0}{\alpha\rho c_p},\ Rn=\frac{R^*}{a},\ \mathrm{Re}_x=\frac{u_w(x)x}{\upsilon},\\&R=\frac{16\sigma^*T_\infty^3}{3\rho c_p},N_b=\frac{\tau D_B(C_w-C_\infty)}{\upsilon},\\&N_t=\frac{\tau D_T(T_w-T_\infty)}{\upsilon T_\infty},\ Gr_x=\frac{g\beta_t(T_w-T_\infty)}{a^2},\\&Gc_x=\frac{g\beta_c(C_w-C_\infty)}{xa^2},\ \varphi=\frac{v^*}{k^*a}\end{aligned} \tag{13}$$

where k is the dimensionless vertex viscosity, λ is the heat generation or absorption term, Rn is the chemical reaction parameter, Gr_x represents the local Grashof number and Gc_x modified local Grashof number, M is the magnetic parameter, R is the radiation parameter.

The transformed boundary conditions are:

$$\begin{aligned}&f(\eta)=s,f'(\eta)=1,\ h(\eta)=0,\ \theta(\eta)=1,\ \phi(\eta)=1,\quad \text{at } \eta=0\\&f'(\eta)\to 0,h(\eta)\to 0,\ \theta(\eta)\to 0,\ \phi(\eta)\to 0\quad \text{as } \eta\to\infty\end{aligned} \tag{14}$$

3 Results

Presented below are the results of this research

4 Discussion

Figure 2 shows a decline in the velocity profile with an increase in the inclination parameter γ which agrees with existing literature to depict a corresponding retardation in the strength of the bouncy force with an increase in the inclination parameter by a factor Cosγ because of thermal variation. The effect of the local modified Grashof number on velocity disruption is depicted in Fig. 3. This parameter does physically influence the kinematic viscosity, length and concentration difference of the fluid. It is clear here that it varies proportionally with the velocity of the fluid. The velocity profile is observed to increase by enhancing the Grashof number Gr in Fig. 4. Literally, increase in Grashof number decreases the viscous force that helps the fluid flow, thereby improving the speed of movement. Figure 5 shows that increasing the material parameter will effect a corresponding increase in the velocity profile.

Figure 6 indicates a decrease in the velocity contour as the suction parameter is enhanced. This implies that increasing the porosity will decrease the permeability and thus increasingly retard the velocity of the flow. Figures 7 and 8 reveal a similar scenario of a downturn of the velocity profiles with every increase in the magnetic parameter. The magnetic field is known to release Lorentz force which retards the motion of the fluid flow and that explains the reason for the behavior of the flow. Figure 9 shows that increasing the thermophoresis factor correspondingly enhance the temperature of the fluid. Thermophoresis does improve the thermal boundary layer of the fluid, thereby making up for the concentration and thermal reduction caused by the Lewis and Prandtl numbers observed from Figs. 10 and 11, respectively.

Figures 12 and 13 depict the influence of Brownian motion on concentration and temperature profiles, respectively. Enhancing the Brownian motion parameter

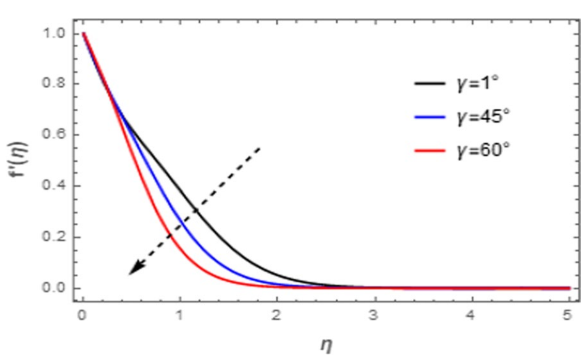

Fig. 2 Influence of γ on the velocity profile

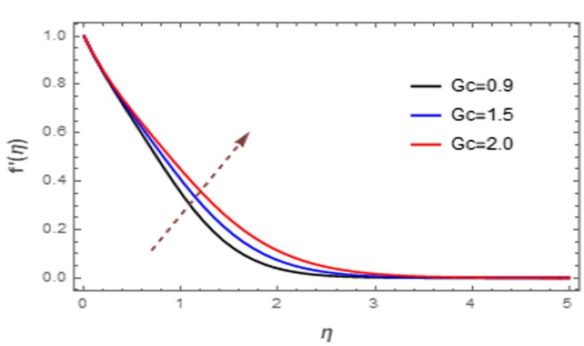

Fig. 3 Influence of Gc on the velocity profile

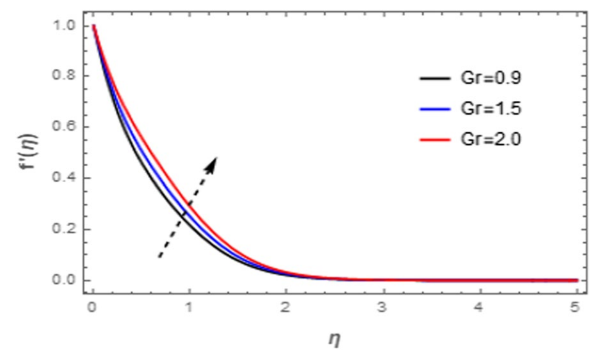

Fig. 4 Influence of Gr on the velocity profile

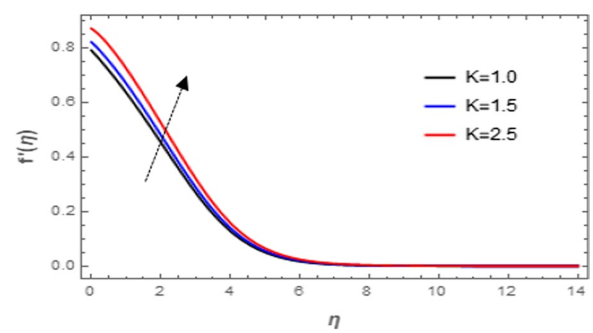

Fig. 5 Influence of K on the velocity profile

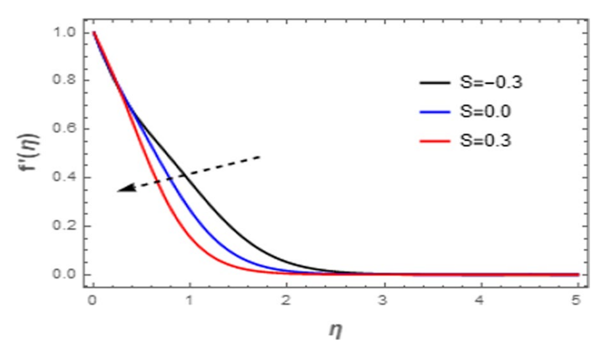

Fig. 6 Influence of S on the velocity profile

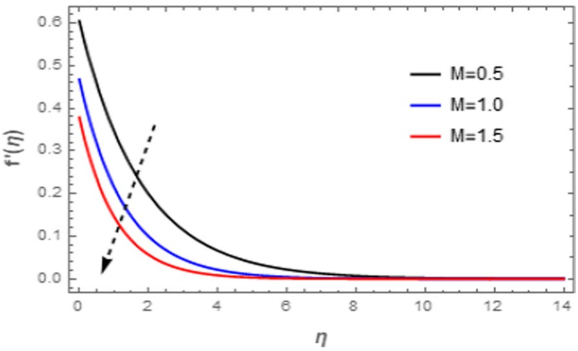

Fig. 7 Influence of M on the velocity profile

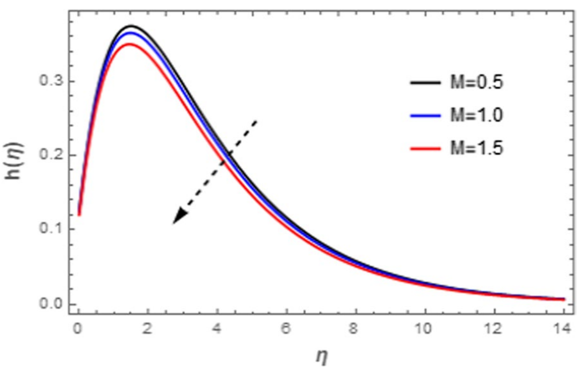

Fig. 8 Influence of M on the angular velocity profile

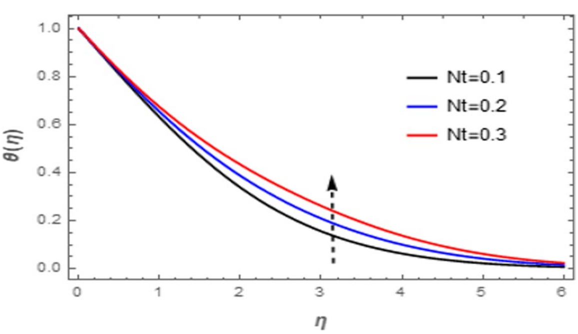

Fig. 9 Influence of Nt on temperature profile

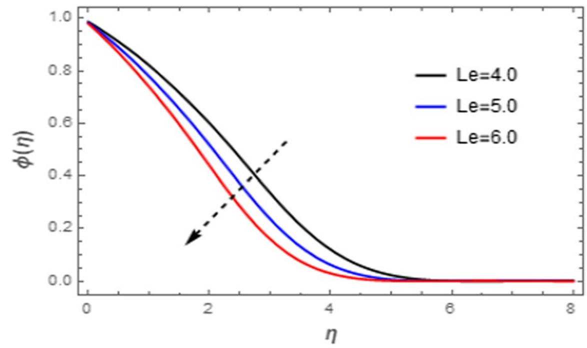

Fig. 10 Influence of Le on concentration profile

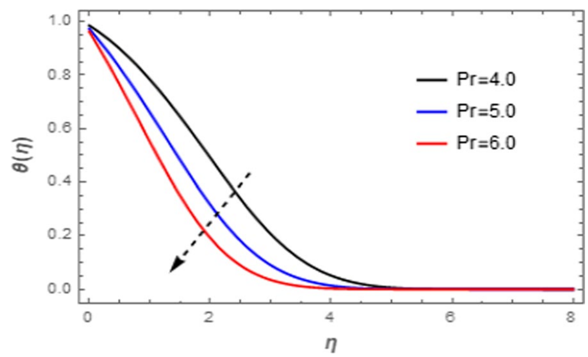

Fig. 11 Influence of Pr on temperature profile

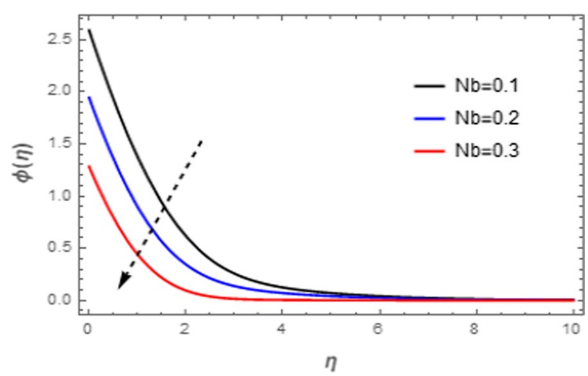

Fig. 12 Influence of Nb on concentration profile

reduces the concentration, while it increases the temperature. Brownian motion brings about randomness and irregularity in the movement of the nanoparticles, this increases the rate of collisions within the flow and thus heats up the thermal boundary layer. This is what causes the decline in the concentration profile. In Fig. 14, an increase is initially observed before it switches to decline in the temperature profile. Figure 15 shows an increase in the angular velocity for every increase in the material parameter.

5 Conclusion

The numerical analysis for the hydromagnetic flow of micropolar nanofluids with co-effects of thermal radiation and chemical reaction over an inclined permeable stretching surface was obtained. The effects of the thermal radiation parameter, magnetic parameter, thermal and solutal Grashof numbers, thermophoresis and Brownian motion parameters among others on the flow were investigated. It was observed that the thermal

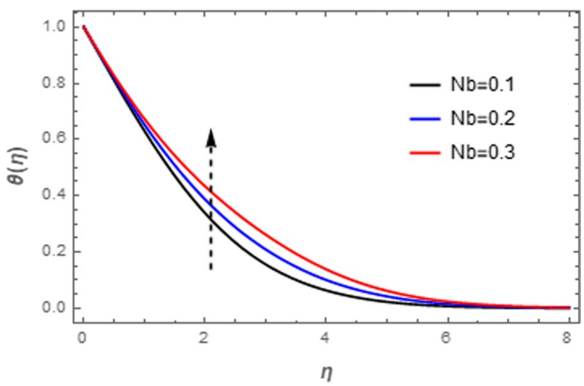

Fig. 13 Influence of Nb on temperature profile

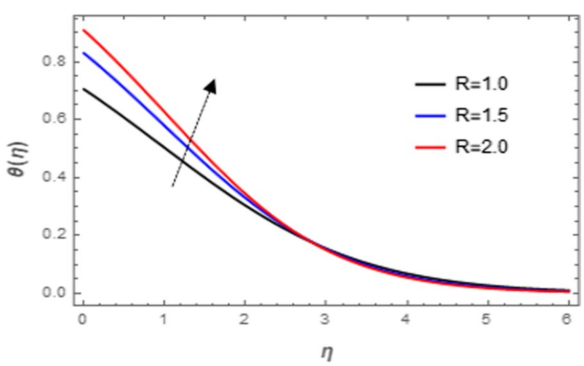

Fig. 14 Influence of R on temperature profile

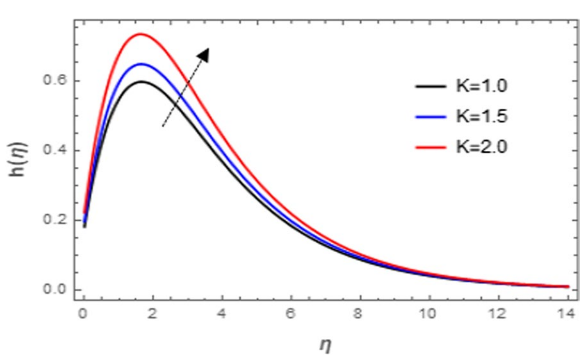

Fig. 15 Influence of K on angular velocity profile

boundary layer increased with an increase in thermal radiation, thermophoresis and Brownian motion parameters. The angular velocity also declined with an increase in the magnetic parameter.

List of symbols

f	Dimensionless stream function
h	Dimensionless angular velocity
T	Temperature
C	Concentration
j	Microinertia density
Pr	Prandtl number
Le	Lewis number
Nb	Brownian motion parameter
Nt	Thermophoresis parameter
Rd	Radiation parameter
K	Material parameter
k_1^*	Vertex viscosity
q_r	Radiative heat flux
t	Time
u	Velocity component in the x—direction
v	Velocity component in the y—direction
x	Cartesian coordinate along the surface
y	Cartesian coordinate normal to the surface
N^*	Angular velocity

Greek symbols

θ	Dimensionless temperature
η	Similarity independent variable
ρ	Density
γ	Angle of inclination
ψ	Stream function
γ^*	Spin gradient velocity
υ	Kinematic viscosity
μ	Dynamic viscosity

Subscripts

w	Condition at the surface
∞	Ambient condition

Superscripts

$'$	Differentiation with respect to η

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Author contributions
AAO contributed to supervision and proof-reading; AP prepared formulation of model and solution.

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Availability of data and materials
The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Ethics approval and consent to participate
Not applicable.

Consent for publication
Not applicable.

Competing interests
The authors declare that they have no competing interests.

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