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Near open generalizations of rough sets and their applications

A. S. Salama[1], A. A. El Atik[1], A. M. Hussein[2], O. A. Embaby[1] and M. S. Bondok[2*]

Abstract

Background The concept of near open sets is a potent tool that empowers researchers to achieve a more encompassing approximation of rough sets, thereby enhancing the accuracy of measurements. The evolution of rough set theory into various generalized forms, based on topological structures, has emerged as a significant approach in the realm of knowledge discovery within databases.

Results This paper's primary contribution lies in the introduction of a novel category of generalized near open sets, termed "inverse simply open sets," within the context of the j-neighborhood space. The paper proposes diverse methods for extending the Pawlak's rough approximations leading to the definition of new approximations in the j-neighborhood space. By employing these newly introduced generalizations, we establish fresh connections between two pivotal theories, namely "general topology and rough set theory". Through a comprehensive investigation, we conduct multiple comparisons between our methodology and classical approaches. Furthermore, we showcase practical applications of these techniques within real-life scenarios, demonstrating their utility in decision-making processes.

Conclusions We reduced the data's ambiguity while increasing its accuracy measure. As a result, we may conclude that the proposed approximations were more precise than earlier techniques and contributed to the elimination of data ambiguity in real-world scenarios requiring accurate decisions.

Keywords Rough sets, Topological space, j-Near open set, j-Neighborhood spaces, b_j*-Open set, j-Inverse simply open sets, Lower and upper approximatons and accuracy measures

1 Introduction

Pawlak's proposal of rough set theory [1, 2] emerged as a valuable tool for addressing the inherent vagueness and uncertainty present in large datasets. This theory is rooted in binary relations, particularly equivalence relations, which can pose challenges due to their inherent restrictions and limitations. Over time, rough set theory has been expanded into various other approaches, some of which have been formulated using topological concepts [3–8]. The notion of topological rough sets, introduced by Wiweger in 1989 [9], stands as a significant topological generalization of rough sets. Extending beyond Pawlak's rough sets, Yao [10] created upper and lower approximations using arbitrary relations without imposing additional conditions on the relations. In order to expand on traditional rough set theory, Abd El-Monsef et al. [11] proposed the concepts of "j-neighborhood space" (abbreviated as j-NS) in 2014. As an extension of open sets into topological spaces, the definition of near open sets was established. Subsequently, W. S. Amer et al. [12] incorporated certain near open set concepts within j-NS frameworks. In 2018, Hosny [13] expanded these estimates to include $\delta\beta$-open and $\wedge_\beta$-open sets. El-Bably [14] employed the concept of "Simply open sets" to extend Pawlak's approximations, employing three distinct methods. The present paper

*Correspondence:
M. S. Bondok
mona.bondok@must.edu.eg
[1] Mathematics Department, Faculty of Science, Tanta University, Tanta, Egypt
[2] Basic Sciences Department, Misr University for Science and Technology, Giza, Egypt

builds upon these advancements, further generalizing these approximations within the context of topological spaces [15–25]. Specifically, the definition of j-near open sets is extended through the introduction of a new category of open sets termed "Inverse simply open sets" within j-neighborhood space. Two distinct methodologies are presented to generalize rough approximation spaces using these inverse simply open sets. The paper proceeds in six sections. In Sect. 2 provides a summary of fundamental concepts. Section 3 introduces the notion of b_j*-open setsand employs them to build the approximations and the properties of these sets are thoroughly examined. Section 4 introduces the concept of j-inverse simply open sets and delves into its properties. In Sect. 5, two distinct methodologies are presented to generalize rough approximation spaces using inverse simply open sets within j-neighborhood space. Section 6 presents an applied example within the domain of plant morphology and Sect. 7 serves as the conclusion of the paper.

2 Preliminaries

This section discusses the fundamental ideas of definitions and properties that will be used in the next sections.

Definition 2.1 [1, 2] For every equivalence relation E on the finite, nonempty set known as the universe U. For each subset $S \subseteq U$, we associate two subsets:

$$\underline{E}(S) = \cup\{T | T \subseteq S, T \in O(U)\}$$

$$\overline{E}(S) = \cap\{T | S \subseteq T, T \in C(U)\}$$

$\overline{E}(S)$ and $\underline{E}(S)$ are the upper and lower approximations of S, respectively and the pair (U, E) is known as an approximation space. The following specifications apply to the pawlak approximation's accuracy and boundary:

$$BN(S) = \overline{E}(S) - \underline{E}(S) \text{and } (S) = \frac{|\underline{E}(S)|}{|\overline{E}(S)|} \text{where} |\overline{E}(S)| \neq 0.$$

Definition 2.2 [26] For any binary relation Q onU. The following are the definitions of the j -neighborhood $p \in \bigcup(N_j(p))$ for each j belongs to $\{r, l, \langle r\rangle, \langle l\rangle, \langle i\rangle, \langle u\rangle, \langle i\rangle, \langle u\rangle\}$:

i. The r-neighborhood when, $N_r(p) = \{r \in U | p\, Q\, r\}$.
ii. The l-neighborhood when, $N_l(p) = \{r \in U | r\, Q\, p\}$.
iii. The ⟨r⟩-neighborhood when, $N_{\langle r\rangle}(p) = \bigcap_{p \in N_r(r)} N_r(r)$.
iv. The ⟨l⟩-neighborhood when, $N_{\langle l\rangle}(p) = \bigcap_{p \in N_l(r)} N_l(r)$.
v. The i-neighborhood when, $N_i(p) = N_r(p) \cap N_l(p)$.
vi. The u-neighborhood when, $N_u(p) = N_r(p) \cup N_l(p)$.
vii. The ⟨i⟩-neighborhood when, $N_{\langle i\rangle}(p) = N_{\langle r\rangle}(p) \cap N_{\langle l\rangle}(p)$.
viii. The ⟨u⟩-neighborhood when, $N_{\langle u\rangle}(p) = N_{\langle r\rangle}(p) \cup N_{\langle l\rangle}(p)$.

Definition 2.3 [26] Assume that (U, Q, ξ j) is a j-neighborhood space with $S \subseteq U$. Subsequently, the j-upper and j-lower approximations (j-positive, j-negative and j -boundary) regions and j-accuracy of $S \subseteq U$ are defined, respectively, as.

- $\overline{Q_j}(S) = \bigcap\{H \in \tau_j : S \subseteq H\} =$ j-closure of S.
- $\underline{Q_j}(S) = \cup\{G \in \tau_j : G \subseteq S\} = j$-interior of S.
- $POS_j(S) = \underline{Q_j}(S)$.
- $NEG_j(S) = U - \underline{Q_j}(S)$.
- $B_j(S) = \overline{Q_j}(S) - \underline{Q_j}(S)$.
- $\sigma_j(S) = \frac{|\underline{Q_j}(S)|}{|\overline{Q_j}(S)|}$, where $|\overline{Q_j}(S)| \neq 0$.

Definition 2.4 [12] Assume that (U, Q, ξ j) is a j-neighborhood space with $S \subseteq U$ is named.

i. j-Regular open if $S = int_j(cl_j(S))$.
ii. j-Pre-open (in brief named P_j-*open*) if $S \subseteq int_j(cl_j(S))$.
iii. j-Semi-open (in brief named S_j-*open*) if $S \subseteq cl_j(int_j(S))$.
iv. γ_j-*open* if $S \subseteq int_j(cl_j(S)) \cup cl_j(int_j(S))$.
v. α_j-*open* if $S \subseteq int_j[cl_j(int_j(S))]$.
vi. β_j-*open* (called semi pre open), if $S \subseteq cl_j[int_j(cl_j(S))]$.

Definition 2.5 [15] Assume that a topological space is (U, τ). The subset S of U is then defined as.

1. A b*-Open set is defined as $S \subseteq cl(int(cl(S))) \cup int(cl(S))$.

2. A b∗-cpen set is defined as $S \supseteq int(cl(int(S))) \cap cl(int(S))$.

The bO ∗ (U) represents the families of all b∗-open sets subsets of a (U, τ), and bC ∗ (U) represents b∗-closed sets.

Definition 2.6 [13] Assume that (U, Q, ξ j) is a j-neighborhood space with $S \subseteq U$. The definition of the subset $\wedge_{\beta_j}(S) = \cap\{G \subseteq U : S \subseteq G,\ G \in \beta_j O(U)\}$. A set S is referred to as a $\wedge_{\beta_j}$-set if S. We refer to the $\vee_{\beta_j}$-set as its complement. The notations $\wedge_{\beta_j}(U)$ and $\vee_{\beta_j}(U)$ represent the families of all $\wedge_{\beta_j}$-sets and $\vee_{\beta_j}$-sets.

Definition 2.7 [13] Assume that $(U,\ Q,\ \xi j)$ is a j-neighborhood space with $S \subseteq U$. Then, the ($\wedge_{\beta_j}$-upper and $\wedge_{\beta_j}$-lower) approximations, ($\wedge_{\beta_j}$-positive, $\wedge_{\beta_j}$-negative, $\wedge_{\beta_j}$-boundary) regions and $\wedge_{\beta_j}$-accuracy measure of S are defined, respectively as follows:

- $\overline{Q}_j^{\wedge\beta}(S) = \cap\left\{H \in \vee_{\beta_j}(U) : S \subseteq H\right\}$.
- $\underline{Q}_j^{\wedge\beta}(S) = \cup\left\{G \in \wedge_{\beta_j}(U) : G \subseteq S\right\}$.
- $POS_j^{\wedge\beta}(S) = \underline{Q}_j^{\wedge\beta}(S)$.
- $POS_j^{\wedge\beta}(S) = \underline{Q}_j^{\wedge\beta}(S)$.
- $B_j^{\wedge\beta}(S) = \overline{Q}_j^{\wedge\beta}(S) - \underline{Q}_j^{\wedge\beta}(S)$.
- $\sigma_j^{\wedge\beta}(S) = \frac{\left|\underline{Q}_j^{\wedge\beta}(S)\right|}{\left|\overline{Q}_j^{\wedge\beta}(S)\right|}, Where\left|\overline{Q}_j^{\wedge\beta}(S)\right|$.

Definition 2.8 [14] Assume that (U, Q, ξ j) is a j-neighborhood space with $S \subseteq U$. Subsequently, ∀ j belongs to $\{r,\ l,\ \langle r\rangle, \langle l\rangle,\ i,\ u,\ \langle i\rangle, \langle u\rangle\}$ if $int_j(cl_j(S)) \subseteq cl_j(int_j(S)$, then the set $S \subseteq U$ is a j-simply open set. A j-simply closed is the complement of a j-simply open sets of $\cup(SM_jO(U))$ represent the famililes of all j-simply open sets and sets of $\cup(SM_jC(U))$ represent the families of all j-simply closed sets.

Definition 2.9 [14] Assume that (U, Q, ξ j) is a j-neighborhood space with $S \subseteq U$. Subsequently, ∀ j belongs to $\{r,\ l,\ \langle r\rangle, \langle l\rangle,\ i,\ u,\ \langle i\rangle, \langle u\rangle\}$, the (j-simply upper and j-simply lower) approximations, (j simply boundary, j-simply positive and j -simply negative) regions and j-simply accuracy measure of $S \subseteq U$ are given, respectively as follows:

- $\underline{Q}_j^{sm}(S) = \cup\{G \in SM_jO(U) :\ G \subseteq\ S\}$.
- $\overline{Q}_j^{sm}(S) = \cap\{H \in SM_jO(U) :\ S\ \subseteq H\}$.
- $B_j^{sm}(S) = \overline{Q}_j^{sm}(S) - \underline{Q}_j^{sm}(S)$.
- $POS_j^{sm}(S) = \underline{Q}_j^{sm}(S)$.
- $NEG_j^{sm}(S) = U - \underline{Q}_j^{sm}(S)$.
- $\sigma_j^{sm}(S) = \frac{\left|\underline{Q}_j^{sm}(S)\right|}{\left|\overline{Q}_j^{sm}(S)\right|}, Where\left|\overline{Q}_j^{sm}(S)\right| \neq 0$.

3 Generalized rough approximations via b_j∗-open sets

The main objective of this section is to provide a novel technique for defining the fundamental ideas of rough sets utilising the notion of b_j∗-open sets.

Definition 3.1 Assume that (U, Q, ξ j) is a j-neighborhood space with $S \subseteq U$. Subsequently, for each j belongs to $\{r,\ l,\ \langle r\rangle, \langle l\rangle,\ i,\ u,\ \langle i\rangle, \langle u\rangle\}$, the set S of U is described as:

1. A b_j∗-Open set if $S \subseteq cl_j(int_j(cl_j(S))) \cup int_j(cl_j(S))$.
2. A b_j∗-closed set if $S \supseteq int_j(cl_j(int_j(S))) \cap cl_j(int_j(S))$.

The families of all b_j∗-Open sets are always represented by $b_j * O(U)$. The complements of b_j∗-Open sets are referred to as "b_j∗-closed sets" and the families of all b_j∗-Closed sets are always represented by $b_j * C(U)$.

Example 3.1 Let U = {x, y, v, w, z} and Q = {(x, x), (x, z), (y, v), (y, w), (y, z), (v, v), (v, w), (w, v), (w, w), (z z)} be a binary relation defined on U. So, xQ = {x, z}, yQ = {v, w, z}, vQ = wQ = {v, w} and zQ = {z}. Consequently, the topology connected to this relation is $\tau_r = \{U, \emptyset, \{z\}, \{x,\ z\}, \{v, w\}, \{v, w\ z\}, \{x, v, w, z\}, \{y, v, w,\ z\}\}$. We will compute the the classes of b_j∗-Open sets for each $j \in r$ as describe:

$$b_r * O(U) = \left\{\begin{array}{c} U, \emptyset, \{v\}, \{w\}, \{z\}, \{x,z\}, \{y,v\}, \{y,w\}, \{y,z\}, \{v,w\}, \{v,z\}, \{w,z\}, \{x,y,z\}, \{x,v,z\}, \{x,w,z\}, \{y,v,w\}, \{y,v,z\}, \{y,w,z\}, \\ \{v,w,z\}, \{x,y,v,z\}, \{x,y,w,z\}, \{x,v,w,z\}, \{y,v,w,z\} \end{array}\right\}$$

.

Proposition 3.1 *Assume that* $(U, Q, \xi j)$ *is a* j*-neighborhood space with* $S \subseteq U$. *Subsequently every* j*-pre open set is* b_j**-open.*

Proof The proof is clear when utilising the definition of b_j*-open set, j-closure characteristics and j-interior characteristics.

Remark 3.1 The previous statement's converse isn't always true as shown in Example 3.1 using topology $\tau_r = \{U, \emptyset, \{z\}, \{x, z\}, \{v, w\}, \{v, w, z\}, \{x, v, w, z\}, \{y, v, w, z\}\}$, $P_rO(U) = \{U, \emptyset, \{x, z\}, \{v, w\}\}$ *and.*

$$b_r * O(U) = \left\{ \begin{array}{c} U, \emptyset, \{v\}, \{w\}, \{z\}, \{x, z\}, \{y, v\}, \{y, w\}, \{y, z\}, \{v, w\}, \{v, z\}, \{w, z\}, \{x, y, z\}, \{x, v, z\}, \{x, w, z\}, \{y, v, w\}, \{y, v, z\}, \{y, w, z\}, \\ \{v, w, z\}, \{x, y, v, z\}, \{x, y, w, z\}, \{x, v, w, z\}, \{y, v, w, z\}\}. \end{array} \right\}$$

It's clear that A subset {v} of U is not j-pre open set, but it is b_r*-Open set.

Lemma 3.1 *Assume that a* j*-approximation is* $(U, Q, \xi j)$. *The following statements are True.*

(1) *The union of* b_j**-Open sets is* b_j**-Open.*
(2) *The intersection of* b_j**-Closed sets is* b_j**-Closed.*

Proof (1) Assume that b_j*-Open sets be a family represented by $\{A_i, i \in I\}$. Then $A_i \subseteq cl_j(int_j(cl_j(A_i))) \cup int_j(cl_j(A_i))$. Hence, $\cup_i A_i \subseteq \cup_i (cl_j(int_j(cl_j(A_i))) \cup int_j(cl_j(A_i)) \subseteq cl_j(int_j(cl_j(\cup_i A_i))) \cup int_j(cl_j(\cup_i A_i)$ for all $i \in I$. b_j*-Open is hence $\cup_i A_i$.

(2) Let be a family of b_j*-Closed represented by $\{A_i, i \in I\}$. Hence, $A_i \supseteq (int_j(cl_j(int_j(A_i))) \cap cl_j(int_j(A_i))$, hence $\cap_i A_i \supseteq \cap_i (int_j(cl_j(int_j(A_i))) \cap cl_j(int_j(A_i))$, $\supseteq (int_j(cl_j(int_j(\cap_i A_i))) \cap cl_j(int_j(\cap_i A_i))$. Therefore $b_j *$ Closed is hence $\cap_i A_i$.

Remark 3.2 Acorrding to Example 3.1, Any two b_j*-Open sets that intersect do not form b_j*-Open sets. Let $S = \{x, y, z\}$ and $T = \{y, v, w\}$ are b_r*-open sets but $S \cap T = \{y\}$ is not b_r*-Open.

Remark 3.3 The families of b_j*-Open sets of U does not form a topology.

Remark 3.4 Assume that $(U, Q, \xi j)$ be a j-neighborhood space with $S \subseteq U$. Subsequently, for each j belongs to $\{r, l, \langle r\rangle, \langle l\rangle, i, u, \langle i\rangle, \langle u\rangle\}$, the following statements listed below aren't always true:

1. $b_u * O(U) \subseteq b_r * O(U) \subseteq b_iO * (U)$.
2. $b_u * O(U) \subseteq b_l * O(U) \subseteq b_iO * (U)$.
3. $b_{\langle u\rangle}{}^*O(U) \subseteq b_{\langle r\rangle}{}^*O(U) \subseteq b_{\langle i\rangle}O^*(U)$.
4. $b_{\langle u\rangle}{}^*O(U) \subseteq b_{\langle l\rangle}{}^*O(U) \subseteq b_{\langle i\rangle}O^*(U)$.
5. The dual of $b_l * O(U)$ is $b_r * O(U)$.
6. The dual of $b_{\langle l\rangle} * O(U)$ is $b_{\langle r\rangle} * O(U)$.

This indicates that the relationships between b_j*-open sets are not comparable as in Example 3.1 We will compute the the classes of b_j*-Open sets for each j belongs to $\{r, l, \langle r\rangle, \langle l\rangle, u, \langle i\rangle, \langle u\rangle\}$ as listed below:

1. $$b_r * O(U) = \left\{ \begin{array}{c} U, \emptyset, \{v\}, \{w\}, \{z\}, \{x, z\}, \{y, v\}, \{y, w\}, \{y, z\}, \{v, w\}, \{v, z\}, \{w, z\}, \{x, y, z\}, \{x, v, z\}, \{x, w, z\}, \{y, v, w\}, \{y, v, z\}, \{y, w, z\}, \\ \{v, w, z\}, \{x, y, v, z\}, \{x, y, w, z\}, \{x, v, w, z\}, \{y, v, w, z\}\} \end{array} \right\}$$

2. $$b_l * O(U) = \left\{ \begin{array}{c} U, \emptyset, \{x\}, \{y\}, \{x, y\}, \{x, z\}, \{y, v\}, \{y, w\}, \{y, z\}, \{x, y, v\}, \{x, y, w\}, \{x, y, z\}, \{y, v, w\}, \{y, v, z\}, \{y, w, z\}, \{x, y, v, w\}, \{x, y, v, z\}, \\ \{x, y, w, z\}, \{y, v, w, z\} \end{array} \right\}$$

3. $b_{u} * O(U) = P(U)$

4. $b_{\langle r\rangle} * O(U) = \left\{ \begin{array}{c} U, \varphi, \{y\}, \{v\}, \{w\}, \{z\}, \{x,z\}, \{y,v\}, \{y,w\}, \{y,z\}, \{v,w\}, \{v,z\}, \{w,z\}, \{x,y,z\}, \{x,v,z\}, \{x,w,z\}, \{y,v,w\}, \{y,v,z\}, \{y,w,z\}, \\ \{v,w,z\}, \{x,y,v,z\}, \{x,y,w,z\}, \{x,v,w,z\}, \{y,v,w,z\} \end{array} \right\}$

5. $b_{\langle l\rangle} * O(U) = \left\{ \begin{array}{c} U, \varphi, \{x\}, \{y\}, \{x,y\}, \{x,z\}, \{y,v\}, \{y,w\}, \{y,z\}, \{x,y,v\}, \{x,y,w\}, \{x,y,z\}, \{y,v,w\}, \{y,v,z\}, \{y,w,z\}, \{x,y,v,w\}, \\ \{x,y,v,z\}, \{x,y,w,z\}, \{y,v,w,z\} \end{array} \right\}$

6. $b_{\langle i\rangle} * O(U) = \left\{ \begin{array}{c} U, \varphi, \{y\}, \{v\}, \{w\}, \{z\}, \{x,z\}, \{y,v\}, \{y,w\}, \{y,z\}, \{v,w\}, \{v,z\}, \{w,z\}, \{x,y,z\}, \{x,v,z\}, \{x,w,z\}, \{y,v,w\}, \{y,v,z\}, \{y,w,z\}, \\ \{v,w,z\}, \{x,y,v,z\}, \{x,y,w,z\}, \{x,v,w,z\}, \{y,v,w,z\} \end{array} \right\}$

7. $b_{\langle u\rangle} * O(U) = \left\{ \begin{array}{c} U, \varphi, \{x\}, \{y\}, \{z\}, \{x,y\}, \{x,z\}, \{y,v\}, \{y,w\}, \{y,z\}, \{x,y,v\}, \{x,y,w\}, \{x,y,z\}, \{y,v,w\}, \{y,v,z\}, \{y,w,z\}, \{x,y,v,w\}, \\ \{x,y,v,z\}, \{x,y,w,z\}, \{y,v,w,z\} \end{array} \right\}$

It is clear that.

1. $b_u{}^*O(U) \subsetneq b_r{}^*O(U)$
2. $b_u{}^*O(U) \subsetneq b_l{}^*O(U)$
3. $b_{\langle u\rangle}{}^*O(U) \subsetneq b_{\langle r\rangle}{}^*O(U)$
4. $b_{\langle u\rangle}O^*(U) \subsetneq b_{\langle l\rangle}{}^*O(U)$
5. $b_{\langle l\rangle}O^*(U) \subsetneq b_{\langle i\rangle}{}^*O(U)$
6. The dual of $b_l * O(U)$ is not $b_r * O(U)$.
7. The dual of $b_r * O(U)$ is not $b_{\langle r\rangle} * O(U)$.

Definition 3.2 Assume that $(U, Q, \xi\ j)$ is a j-neighborhood space with$S \subseteq U$. Subsequently for each j belongs to $\{r, l, \langle r\rangle, \langle l\rangle, i, u, \langle i\rangle, \langle u\rangle\}$. Then.

1. The b_j*-lower approximations of S(called b_j*-interior of S) are the union of all b_j*-Open sets of U contained in the set S, they are represented by the symbol b_j*-int(S).

- $\underline{R}_j^{b^*}(S) = \cup\{G \in b_jO^*(U) : G \subseteq S\} = b_j * int(S)$.

2. The b_j*-upper approximations of S(called b_j*-closure of S) is the intersection of all b_j*-closed sets of U included in S, it is represented by the symbol b_j*-Cl(S).

- $\overline{R}_j^{b^*}(S) = \cap\{H \in b_jC * (U) : S \subseteq H\} = b_j * cl(S)$.

Furthermore, the approximations of S's (b_j*-positive, b_j*-negative, b_j*-boundary) regions and b_j*-accuracy are defined, respectively:

- $POS_j^{b^*}(S) = \underset{-j}{R}^{b^*}(S)$.
- $NEG_j^{b^*}(S) = U - \underset{-j}{R}^{b^*}(S)$.
- $B_j^{b^*}(S) = \overline{R}_j^{b^*}(S) - \underset{-j}{R}^{b^*}(S)$.
- $\sigma_j^{b^*}(S) = \frac{\left|\underline{R}_j^{b^*}(S)\right|}{\left|\overline{R}_j^{b^*}(S)\right|}, Where \quad \left|\overline{R}_j^{b^*}(S)\right| \neq 0$.

The following properties demonstrate the relationships between b_j*-open set approximations and other approximation types.

Proposition 3.2 *Assume that* $(U, Q, \xi\ j)$ *is a* j*-neighborhood space with* $S \subseteq U$. *Then.*

$\underline{Q}_j(S) \subseteq \underset{-j}{R}^{b^*}(S)$.

$\overline{R}_j^{b^*}(S) \subseteq \overline{Q}_j(S)$.

Corollary 3.1 *Assume that* (U, Q, ξ j) *is a* j-*neighborhood space with* S ⊆ U. *Then.*

$B_j^{b^*}(S) \subseteq B_j(S)$.

$\sigma_j(S) \leq \sigma_j^{b^*}(S)$.

The proposal proposition investigates the main characteristics of the b_j* approximations.

Proposition 3.3 *Assume that* (U, Q, ξ j) *be a* j-*neighborhood space and*S, T ⊆ U. *Therefore for each* j *belongs to* $\{r, l, \langle r\rangle, \langle l\rangle, i, u, \langle i\rangle, \langle u\rangle\}$, *the following characteristics hold*:

(L1) $\underline{R}_j^{b^*}(S) \subseteq S$.

(L2) $\underline{R}_j^{b^*}(\varnothing) = \varnothing$.

(L3) $\underline{R}_j^{b^*}(U) = U$.

(L4) $\underline{R}_j^{b^*}(S \cap T) \subseteq \underline{R}_j^{b^*}(S) \cap \underline{R}_j^{b^*}(T)$.

(U4) $\overline{R}_j^{b^*}(S \cup T) \supseteq \overline{R}_j^{b^*}(S) \cup \overline{R}_j^{b^*}(T)$.

(L5) If S ⊆ T, then $\underline{R}_j^{b^*}(S) \subseteq \underline{R}_j^{b^*}(T)$.

(L6) $\underline{R}_j^{b^*}(S) \cup \underline{R}_j^{b^*}(T) \subseteq \underline{R}_j^{b^*}(S \cup T)$.

(L7) $\underline{R}_j^{b^*}\left(\underline{R}_j^{b^*}(S)\right) = \underline{R}_j^{b^*}(S)$.

(L8) $\underline{R}_j^{b^*}(S) = (\overline{R}_j^{b^*}(S^c))^c$.

(L9) $\underline{R}_j^{b^*}(\underline{R}_j^{b^*}(S)) \subseteq \overline{R}_j^{b^*}(\underline{R}_j^{b^*}(S))$.

(L10) $x \in \underline{R}_j^{b^*}(S) \leftrightarrow \exists\, G \in b_j * O(U),\ x \in G,\ G \subseteq S$.

(U1) $S \subseteq \overline{R}_j^{b^*}(S)$

(U2) $\overline{R}_j^{b^*}(\varnothing) = \varnothing$.

(U3) $\overline{R}_j^{b^*}(U) = U$.

(U5) If S ⊆ T, then $\overline{R}_j^{b^*}(S) \subseteq \overline{R}_j^{b^*}(T)$.

(U6) $\overline{R}_j^{b^*}(S) \cap \overline{R}_j^{b^*}(T) \supseteq \overline{R}_j^{b^*}(S \cap T)$.

(U7) $\overline{R}_j^{b^*}\left(\overline{R}_j^{b^*}(S)\right) = \overline{R}_j^{b^*}(S)$.

(U8) $\overline{R}_j^{b^*}(S) = (\underline{R}_j^{b^*}(S^c))^c$.

(U9) $\overline{R}_j^{b^*}\left(\overline{R}_j^{b^*}(S)\right) \supseteq \underline{R}_j^{b^*}\left(\overline{R}_j^{b^*}(S)\right)$.

(U10) $x \in \overline{R}_j^{b^*}(S) \leftrightarrow G \cap S \neq \varnothing, \forall G \in b_j * O(U),\ x \in G$.

Proof The proof is clear when utilizing the characteristics of b_j*-interior and b_j*-closure.

Definition 3.3 Assume that (U, Q, ξ j) be a j-neighborhood space with S ⊆ U and for each j belongs to $\{r, l, \langle r\rangle, \langle l\rangle, i, u, \langle i\rangle, \langle u\rangle\}$. The set S is named:

i. b_j*-definable (b_j*-Exact) if $\underline{R}_j^{b^*}(S) = \overline{R}_j^{b^*}(S)$ or $B_j^{b^*}(S) = \emptyset$.
ii. b_j*-rough if $\underline{R}_j^{b^*}(S) \neq \overline{R}_j^{b^*}(S)$ or $B_j^{b^*}(S) \neq \emptyset$.

Remark 3.5 Assume that (U, Q, ξ j) be a j-neighborhood space andS, T ⊆ U. Any two b_j*-exact sets do not always have to intersect to be b_j*-exact sets. Acorrding to Example 3.1, let S = {x, y, z } and T = {y, v, w} are b_r*-exact sets but S ∩ T = {y} is not b_r*-exact sets.

Corollary 3.2 *Assume that* (U, Q, ξ j) *is a* j-*neighborhood space with*S ⊆ U. *Subsequently for each* j *belongs to* $\{r, l, \langle r\rangle, \langle l\rangle, i, u, \langle i\rangle, \langle u\rangle\}$. *If* S *is a* j-*exact set implies to* S *is* b_j*-*exact sets.*

Remark 3.6 The converse of Corollary 3.2 does not always hold. According to Example 3.1, S = {x, w} is a b_r*-exact sets, but it is rough.

4 J-inverse simply open sets

Now we will study the definition of inverse simply open sets defined in the j-Neighborhood Space, review the basic properties and some important theorems.

Definition 4.1 Assume that $(U, Q, \xi j)$ be a j-neighborhood space. Subsequently for each j belongs to $\{r, l, \langle r \rangle, \langle l \rangle, i, u, \langle i \rangle, \langle u \rangle\}$. The set $S \subseteq U$ is named j -inverse simply open set if $cl_j(int_j(S)) \subseteq int_j(cl_j(S))$.

Remark 4.1

1. $ISM_jO(U)$ is the family of j-inverse simply open sets of U.
2. The "j-inverse simply closed" is the complement of a j-inverse simply open and $ISM_jC(U)$ represents the family of j-inverse simply closed sets of U.

Theorem 4.1 *Assume that* $(U, Q, \xi j)$ *be a* j*-neighborhood space. Subsequently, for each* j *belongs to* $\{r, l, \langle r \rangle, \langle l \rangle, i, u, \langle i \rangle, \langle u \rangle\}$, *the collections of* $ISM_jO(U)$ *are a topology on the universal*U.

Proof i. $\varnothing$ and U are j-inverse simply open set.

ii. Let$\{S_i | i \in I\} \in ISM_jO(U)$.Then, $cl_j(int_j(S_i)) \subseteq int_j(cl_j(S_i)) \forall i \in I$ and $\cup_i cl_j(int_j(S_i)) \subseteq \cup_i int_j(cl_j(S_i))$ this implies to $\cup_i cl_j(int_j(S_i)) = cl_j(int_j(\cup_i S_i))$ and $\cup_i int_j(cl_j(S_i)) = int_j(cl_j(\cup_i S_i))$ therefore $cl_j(int_j(\cup_i S_i)) \subseteq int_j(cl_j(\cup_i S_i))$ and thus $\cup_i S_i$ is a j -inverse simply open.

iii. Let$S_1, S_2 \in ISM_jO(U)$, then $cl_j(int_j(S_1)) \subseteq int_j(cl_j(S_1))$ and $cl_j(int_j(S_2)) \subseteq int_j(cl_j(S_2))$. Then $int_j(cl_j(S_1 \cap S_2)) \subseteq cl_j(int_j S_1 \cap S_2)$. Thus $S_1 \cap S_2$ is a j-inverse simply open.

From (i), (ii) and (iii) $ISM_jO(U)$ is a topology on U.

Theorem 4.2 *Assume that* $(U, Q, \xi j)$ *be a* j*-neighborhood space. Every* j*-inverse simply open set is correspondingly a* j*-inverse simply closed set and vice versa.*

Proof Assume that S be a j-inverse simply open set. So, $cl_j(int_j(S)) \subseteq int_j(cl_j(S))$. By taking the complement of the both sides, we obtain: $[cl_j(int_j(S))]^c \supseteq [int_j(cl_j(S))]^c$ and this implies to $int_j[int_j(S))^c] \supseteq cl_j[cl_j(S))^c]$.Therefore $cl_j(int_j(S^c)) \subseteq int_j(cl_j(S^c))$ and S^c is j-inverse simply open.

Corollary 4.1 *Assume that* $(U, Q, \xi j)$ *be a j-NS and for each* j *belongs to* $\{r, l, \langle r \rangle, \langle l \rangle, i, u, \langle i \rangle, \langle u \rangle\}$. *Then,* $ISM_jO(U) = ISM_jC(U)$ *and each of these topologies are quasi-discrete.*

Remark 4.2 Assume that $(U, Q, \xi j)$ is a j-neighborhood space with $S \subseteq U$ and for each j belongs to $\{r, l, \langle r \rangle, \langle l \rangle, i, u, \langle i \rangle, \langle u \rangle\}$. Then, In general, the following statements are untrue:

1. $ISM_uO(U) \subseteq ISM_rO(U) \subseteq ISM_iO(U)$.
2. $ISM_uO(U) \subseteq ISM_lO(U) \subseteq ISM_iO(U)$.
3. $ISM_{\langle u \rangle}O(U) \subseteq ISM_{\langle r \rangle}O(U) \subseteq ISM_{\langle i \rangle}O(U)$.
4. $ISM_{\langle u \rangle}O(U) \subseteq ISM_{\langle l \rangle}O(U) \subseteq ISM_{\langle i \rangle}O(U)$.
5. The dual of $ISM_lO(U)$ is $ISM_rO(U)$.
6. The dual of $ISM_{\langle r \rangle}O(U)$ is $ISM_{\langle l \rangle}O(U)$.

The example that follows demonstrates Remark 4.2

Example 4.1 According to Examle 3.1 We will compute the topology associated with this relation and the families of al of j-inverse simply closed setsand j-inverse simply open sets for each j belongs to $\{r, l, \langle r \rangle, \langle l \rangle, i, u, \langle i \rangle, \langle u \rangle\}$ as described:

1. $ISM_rO(U) = \left\{ \begin{array}{c} U, \varnothing, \{x\}, \{y\}, \{v\}, \{w\}, \{x, y\}, \{x, v\}, \{y, v\}, \{x, w\}, \{y, w\}, \{v, z\}, \{w, z\}, \{x, y, v\}, \{x, y, w\}, \{x, v, z\}, \{y, v, z\}, \{x, w, z\}, \\ \{y, w, z\}, \{v, w, z\}, \{x, y, v, z\}, \{x, y, w, z\}, \{x, v, w, z\}, \{y, v, w, z\} \end{array} \right\}$
2. $ISM_lO(U) = \left\{ \begin{array}{c} U, \varnothing, \{v\}, \{w\}, \{z\}, \{x, y\}, \{v, w\}, \{v, z\}, \{w, z\}, \{x, y, v\}, \{x, y, w\}, \{x, y, z\}, \{v, w, z\}, \{x, y, v, w\}, \{x, y, v, z\}, \{x, y, w, z\}, \\ \{y, v, w, z\} \end{array} \right\}$

3. $ISM_{i}O(U) = P(U)$.
4. $ISM_{u}O(U) = P(U)$.
5. $ISM_{\langle r\rangle}O(U) = P(U)$.
6. $ISM_{\langle l\rangle}O(U) = \{U, \emptyset, \{v\}, \{w\}, \{z\}, \{x, y\}, \{v, w\}, \{v, z\}, \{w, z\}, \{x, y, v\}, \{x, y, w\}, \{x, y, z\}, \{v, w, z\}, \{x, y, v, w\}, \{x, y, v, z\}, \{x, y, w, z\}\}$
7. $ISM_{\langle i\rangle}O(U) = P(U)$.
8. $ISM_{\langle u\rangle}O(U) = P(U)$.

The previous results show that:

- $ISM_{u}O(U) \subsetneq ISM_{r}O(U)$
- $ISM_{u}O(U) \subsetneq ISM_{l}O(U)$.
- $ISM_{\langle u\rangle}O(U) \subsetneq ISM_{\langle l\rangle}O(U)$.
- $ISM_{r}O(U)$ is not the dual of $ISM_{l}O(U)$.
- $ISM_{\langle r\rangle}O(U)$ is not the dual of $ISM_{\langle l\rangle}O(U)$.

Example 4.1 shows that the linkages between ISM_j-open setsfor various kinds of τ_j are independent.

5 Generalizions of j-inverse simply open sets

In the following section, we present two distinct strategies for generalizing Pawlak rough set approximations in terms of topological spaces. The offered strategies are extremely beneficial in real-world applications and play a vital role in decision-making.

Definition 5.1 Assume that $(U, Q, \xi j)$ is a j-neighborhood spacewith $S \subseteq U$. Then, for each j belongs to $\{r, l, \langle r\rangle, \langle l\rangle, i, u, \langle i\rangle, \langle u\rangle\}$. The ISM_j–lower approximations,ISM_j-upperapproximations,ISM_j-boundary,ISM_j-positive –regions, ISM_j–negative regions and the ISM_j-accuracy of S are described as follows:

- $\underline{R}_{j}^{Ism}(S) = \bigcup\{G \in ISM_jO(U)) : G \subseteq S\}$ called ISM_j-interior of S.
- $\overline{R}_{j}^{Ism}(S) = \bigcap\{H \in ISM_jC(U) : S \subseteq H\}$ called ISM_j-closure of S.
- $B_{j}^{Ism}(S) = \overline{R}_{j}^{Ism}(S) - \underset{-j}{R}^{Ism}(S)$.
- $POS_{j}^{Ism}(S) = \underset{-j}{R}^{Ism}(S)$.
- $NEG_{j}^{Ism}(S) = U - \overline{R}_{j}^{Ism}(S)$.
- $\sigma_{j}^{Ism}(S) = \frac{\left|\underset{-j}{R}^{Ism}(S)\right|}{\left|\overline{R}_{j}^{Ism}(S)\right|}$, Where $\left|\overline{R}_{j}^{Ism}(S)\right| \neq 0$.

Definition 5.2 Assume that $(U, Q, \xi j)$ is a j-neighborhood spacewith$S \subseteq U$. For each j belongs to $\{r, l, \langle r\rangle, \langle l\rangle, i, u, \langle i\rangle, \langle u\rangle\}$. A set S is named:

i. ISM_j − exact if $\underset{-j}{R}^{Ism}(S) = \overline{R}_{j}^{Ism}(S)$ or $B_{j}^{Ism}(S) = \emptyset$ and $\sigma_{j}^{Ism}(S) = 1$.
ii. ISM_j − rough if $\underset{-j}{R}^{Ism}(S) \neq \overline{R}_{j}^{Ism}(S)$ or $B_{j}^{Ism}(S) \neq \emptyset$.

The proposition proposal investigates the key characteristics of the current ISM_j − upper, and ISM_j-lower approximations.

Proposition 5.1 *Assume that* $(U, Q, \xi j)$ *is a* j*-neighborhood spacewith*$S, T \subseteq U$. *For each* j *belongs to* $\{r, l, \langle r\rangle, \langle l\rangle, i, u, \langle i\rangle, \langle u\rangle\}$. *The properties holds*:

(L1) $\underset{-j}{R}^{Ism}(S) \subseteq S$.

(L2) $\underset{-j}{R}^{Ism}(\emptyset) = \emptyset$.

(L3) $\underset{-j}{R}^{Ism}(U) = U$.

(L4) $\underline{R}_{j}^{Ism}(S \cap T) = \underline{R}_{j}^{Ism}(S) \cap \underline{R}_{j}^{Ism}(T)$.

(L5) If$S \subseteq T$, then $\underset{-j}{R}^{Ism}(S) \subseteq \underset{-j}{R}^{Ism}(T)$.

(L6) $\underset{-j}{R}^{Ism}(S) \cup \underset{-j}{R}^{Ism}(T) \subseteq \underset{-j}{R}^{Ism}(S \cup T)$.

(L7) $\underset{-j}{R}^{Ism}(S^c) = (\overline{R}_{j}^{Ism}(S))^c$.

(L8) $\underset{-j}{R}^{Ism}\left(\underset{-j}{R}^{Ism}(S)\right) = \underset{-j}{R}^{Ism}(S)$.

(L9) $\underset{-j}{R}^{Ism}((\underset{-j}{R}^{Ism}(S))^c) = (\underset{-j}{R}^{Ism}(S))^c$.

(L10) $\underset{-j}{R}^{Ism}(S) = \overline{R}_{j}^{Ism}\left(\underset{-j}{R}^{Ism}(S)\right)$.

(L11) $\forall$ x $\in$ $ISM_j O(U) \rightarrow \underline{R}_{j}^{Ism}(X) = X$.

(U1) $S \subseteq \overline{R}_j^{Ism}(S)$.

(U2) $\overline{R}_j^{Ism}(\varnothing) = \varnothing$.

(U3) $\overline{R}_j^{Ism}(U) = U$.

(U4) $\overline{R}_j^{Ism}(S \cup T) = \overline{R}_j^{Ism}(S) \cup \overline{R}_j^{Ism}(T)$.

(U5) If$S \subseteq T$, then $\overline{R}_j^{Ism}(S) \subseteq \overline{R}_j^{Ism}(T)$.

(U6) $\overline{R}_j^{Ism}(S) \cap \overline{R}_j^{Ism}(S) \supseteq \overline{R}_j^{Ism}(S \cap T)$.

(U7) $\overline{R}_j^{Ism}(S^c) = (\underline{R}_{j}^{Ism}(S))^c$.

(U8) $\overline{R}_j^{Ism}\left(\overline{R}_j^{Ism}(S)\right) = \overline{R}_j^{Ism}(S)$.

(U9) $\overline{R}_j^{Ism}((\overline{R}_j^{Ism}(S))^c) = \overline{R}_j^{Ism}(S))^c$.

(U10) $\overline{R}_j^{Ism}(S) = \underline{R}_{j}^{Ism}\left(\overline{R}_j^{Ism}(S)\right)$.

(U11) $\forall x \in ISM_j O(U) \rightarrow \overline{R}_j^{Ism}(X) = X$.

Proof The characteristics from (U1–U11) and (L1–L11) are thus satisfied by applying the properties of of ISM_j -interior and ISM_j-closure of S.

Definition 5.3 Assume that $(U, Q, \xi j)$ is a j-neighborhood space with$S \subseteq U$. Then, for each j belongs to $\{r, l, \langle r\rangle, \langle l\rangle, i, u, \langle i\rangle, \langle u\rangle\}$. The M_j-upper approximation,M_j-lower approximations,M_j-boundary regions,M_j -positive regions, M_j-negative regions and M_j-accuracy of S are given, respectively, by:

- $\overline{M}_j(S) = \overline{R}_j^{Ism}(S) \cap \overline{R}_r^{b^*}(S)$.
- $\underline{M}_{j}(S) = \underline{R}_{j}^{Ism}(S) \cup \underline{R}_{r}^{b^*}(S)$.
- $B_j^M(S) = \overline{M}_j(S) - \underline{M}_{j}(S)$.
- $POS_j^M(S) = \underline{M}_{j}(S)$.
- $NEG_j^M(S) = U - \overline{M}_j(S)$.
- $\sigma_j^M(S) = \frac{|\underline{M}_{j}(S)|}{|\overline{M}_j(S)|}$, Where $|\overline{M}_j(S)| \neq 0$.

Definition 5.4 Assume that $(U, Q, \xi j)$ is a j-neighborhood space with$S \subseteq U$. For each j belongs to $\{r, l, \langle r\rangle, \langle l\rangle, i, u, \langle i\rangle, \langle u\rangle\}$. A set S is named:

1. Exact if $\underline{M}_{j}(S) = \overline{M}_j(S)$ or $B_j^M(S) = \varnothing$ and $\sigma_j^M(S) = 1$.
2. R ough if $\underline{M}_{j}(S) \neq \overline{M}_j(S)$ or $B_j^M(S) \neq \varnothing$.

The relationships between M_j-approximations and some of the other approximation types are illustrated by the following properties.

Proposition 5.2 *Assume that* $(U, Q, \xi j)$ *be a* j*-neighborhood space with* $S \subseteq U$. *Then.*

$$\underline{R}_j(S) \subseteq \underline{M}_j(S).\overline{M}_j(S) \subseteq \overline{R}_j(S).$$

$$\underline{R}_j^{Ism}(S) \subseteq \underline{M}_j(S).\overline{M}_j(S) \subseteq \overline{R}_j^{Ism}(S).$$

Proof:is obvious.

Corollary 5.1 *Assume that* $(U, R, \xi j)$ *be a* j*-neighborhood space with* $S \subseteq U$. *Then.*

$$B_j^M(S) \subseteq B_j(S).\sigma_j(S) \leq \sigma_j^M(S).$$

$$B_j^M(S) \subseteq B_j^{Ism}(S).\sigma_j^{Ism}(S) \leq \sigma_j^M(S).$$

Proof: is obvious.

Corollary 5.2 *Assume that* $(U, Q, \xi j)$ *is* a j*-neighborhood space with* $S \subseteq U$. *Then, if* S *is j-exact* $\rightarrow$ S *is j-inverse simply exact* $\rightarrow$ S *is* M_j*-exact.*

Remark 5.1 The reverse of the corollary 5.2 is not right in overall as shown in Example 5.1

Example 5.1 The class of all $\wedge_{\beta_r}$-open sets and $\vee_{\beta_r}$ -closed sets are displayed in Example 3.1, respectively:

$$\wedge_{\beta_r} O(U) = \left\{ \begin{array}{c} U, \emptyset, \{y\}, \{v\}, \{w\}, \{z\}, \{x,z\}, \{y,v\}, \{y,w\}, \{y,z\}, \{v,w\}, \{v,z\}, \{w,z\}, \{x,y,z\}, \{x,v,z\}, \{x,w,z\}, \{y,v,w\}, \{y,v,z\}, \{y,w,z\}, \\ \{v,w,z\}, \{x,y,v,z\}, \{x,y,w,z\}, \{x,v,w,z\}, \{y,v,w,z\} \end{array} \right\}$$

$$\vee_{\beta_r} O(U) = \left\{ \begin{array}{c} U, \emptyset, \{x\}, \{y\}, \{v\}, \{w\}, \{x,y\}, \{x,v\}, \{x,w\}, \{x,z\}, \{y,v\}, \{y,w\}, \{v,w\}, \{x,y,v\}, \{x,y,w\}, \{x,y,z\}, \{x,v,w\}, \{x,v,z\}, \{x,w,z\}, \\ \{y,v,w\}, \{x,y,v,w\}, \{x,y,v,z\}, \{x,y,w,z\}, \{x,v,w,z\} \end{array} \right\}$$

Table 1 Comparison between the boundary regions and accuracy approximations for j ∈ r

P(U)	M. E. Abd El-Monsef		M. Hosny		M.K. El-Bably		Ism-generalization		M-generalization	
	$B_r(S)$	$\sigma_r(S)$	$B_r^{\wedge\beta}(S)$	$\sigma_r^{\wedge\beta}(S)$	$B_r^{sm}(S)$	$\sigma_r^{sm}(S)$	$B_r^{Ism}(S)$	$\sigma_r^{Ism}(S)$	$B_r^{M}(S)$	$\sigma_r^{M}(S)$
{x}	{x}	0	{x}	0	∅	1	{x}	1	∅	1
{y}	{y }	0	∅	1	∅	1	{y }	1	∅	1
{v}	{y, v, w}	0	∅	1	{v, w}	0	∅	1	∅	1
{w}	{y, v, w}	0	∅	1	{v, w}	0	∅	1	∅	1
{z}	{x, y}	1/3	{x}	½	∅	1	{x}	0	∅	1
{x, y}	{x, y}	0	{x}	1/2	∅	1	{x, y }	1	∅	1
{x, v}	{x, y, v, w}	0	{x}	1/2	{v, w}	1/3	{x}	1	∅	1
{x, w}	{x, y , v, w}	0	{x}	1/2	{v, w}	1/3	{x}	1	∅	1
{x, z}	{y }	2/3	∅	1	∅	1	∅	½	∅	1
{y , v}	{y , v, w}	0	∅	1	{v, w}	1/3	∅	1	∅	1
{y , w}	{y , v, w}	0	∅	1	{v, w}	1/3	∅	1	∅	1
{y , z}	{x, y }	1/3	{x}	2/3	∅	1	{x}	½	∅	1
{v, w}	{y }	2/3	∅	1	∅	1	∅	2/3	∅	1
{v, z}	{x, y , v, w}	1/5	{x}	2/3	{v, w}	1/3	{x}	1	∅	1
{w, z}	{x, y , v, w}	1/5	{x}	2/3	{v, w}	1/3	{x}	1	∅	1
{x, y, v}	{x, y , v, w}	0	{x}	2/3	{v, w}	½	{x}	1	∅	1
{x, y, w}	{x, y, v, w}	0	{x}	2/3	{v, w}	½	{x}	1	∅	1
{x, , z}	{y}	2/3	∅	1	∅	1	∅	¾	∅	1
{x, v, w}	{x, y}	1/2	{x}	¾	∅	1	{x}	2/3	∅	1
{x, v, z}	{y , v, w}	2/5	∅	1	{v, w}	½	∅	1	∅	1
{x, w, z}	{y , v, w}	2/5	∅	1	{v, w}	½	∅	1	∅	1
{y, v, w}	{x, y }	1/2	∅	1	∅	1	∅	¾	∅	1
{y, v, z}	{x, y , v, w}	1/5	{x}	3/4	{v, w}	½	{x}	1	∅	1
{y, w, z}	{x, y , v, w}	1/5	{x}	¾	{v, w}	½	{x}	1	∅	1
{v, w, z}	{x, y }	3/5	{x}	¾	∅	1	{x, y }	1	∅	1
{x, y , v, w}	{x,y}	1/2	{x}	¾	∅	1	{x}	4/5	∅	1
{x, y, v, z}	{y , v, w}	2/5	{x}	¾	{v, w}	3/5	∅	1	∅	1
{x, y, w, z}	{y , v, w}	2/5	{x}	¾	{v, w}	3/5	∅	1	∅	1
{x, v, w, z}	{y }	4/5	∅	1	∅	1	{y }	1	∅	1
{y, v, w, z}	{x}	4/5	∅	1	∅	1	{x}	1	∅	1
U	∅	1	∅	1	∅	1	∅	1	∅	1

As shown in Table 1, we use M. E. Abd El-Monsef in Definition 2.3, M. Hosny in Definition 2.7, M.K. El-Bably in Definition 2.9, Definition 5.1, and Definitions 5.3 to calculate the boundary region and approximation accuracy.

Comparison between M. E. Abd El-Monsef in Definition 2.3, M. Hosny in Definition 2.7, M.K. El-Bably in Definition 2.9, Definition 5.1, and Definitions 5.3 respectively.

From the above comparison, we can see that the current methods are more accurate than prior ways and minimise the boundary region, which is highly important in the rough set context of reducing ambiguity.

6 Plant morphology application

The main goal of this section is to provide real-world examples that highlight the importance of using the proposed approaches in the context of rough sets.

6.1 Description

Morphology serves as the foundation for categorizing plants into distinct types. The section introduces a methodology for quantifying the similarity between various aspects of plant morphology, specifically focusing on three key attributes: (1) plant topological structure which describe the structural relationship between various organs, (2) the peripheral outlines of a plant and the contour of each branch and (3) the inner features which describe the geometric characteristics such as branching angles and diameters of the different organs. The topological structures are described using tree graphs and their similarity between each pair of trees can be calculated based on the cost of transformation between two graphs using the edit distance of these graphs and branch degradation. For the two tree graphs, let one tree be the source tree T_s and the other be the target tree T_d. After the branch degradation on both T_d and T_s, we perform certain insertions and deletions to transform T_s to T_d. The total cost $D_t(T_s, T_d)$ of the transformation is defined recursively as follows

$$D_t(T_s, T_d) = D_t(T[p(T_s)], (T[p(T_d)]) + \sum_{i=1}^{\max\{|b(T_s)|,|b(T_d)|\}} D_t(T[b(T_s, i)], T[b(T_d, i)]) \quad (1)$$

where all of the root node's axial successor nodes excluding branching nodes are represented by the symbol p(T). All of the root node's branching nodes are represented by b(T). The ith node in this set is represented by b(T, i). After computing the cost recursively using Eq. (1), we may obtain the topological structure similarity value (which ranges from 0 to 1) with 1 denoting the maximum similarity by employing linear transformations to clamp the cost computation result within the range between 0 and 1.The transformation equation is defined as follows:

$$S_t(T_s, T_d) = 1 - \frac{D(T_s, T_d) + ||T_s| - |T_d||}{2\,\text{MAX}\,(|T_s|, |T_d|)} \quad (2)$$

Note that: The presented application is constructed using practical issues found in [27]. The data were analyzed, and the results were used to determine the tree topologies.

6.2 Assumption

Let $U = \{T_1, T_2, T_3, T_4, T_5\}$ be a different 5 theoretical plants. Every theoretical plant is composed of elementary entities. Their entities connections are not organised the same way. The similarities between each pair are calculated by using Eq. (2) and the results of our calculation of the similarities between each pair of trees are shown in Table 2. The values range from 0 to 1 with 1 indicating exactly the same between two tree structures.

We consider the relation on U which represent high similarity defined as follow:$R = \left\{\left(T_I, T_j\right) : S_t\left(T_I, T_j\right) \geq 0.70,\ i, j = 1,2,3,4,5\right\}$,Then $T_I R T_j = \{(T_1, T_1), (T_1, T_5), (T_2, T_2), (T_2, T_3), (T_2, T_4), (T_2, T_5), (T_3, T_3), (T_3, T_4), (T_4, T_3), (T_4, T_4), (T_5, T_1), (T_5, T_5)\}$.

The following are the right neighborhoods for each element of U in reference to relation R:

$T_1R = \{T_1, T_5\}$, $T_2R = \{T_2, T_3 T_4, T_5\}$, $T_3R = \{T_3, T_4\}$, $T_4R = \{T_3, T_4\}$ and $T_5R = \{T_1, T_5\}$.

Consequently, the topology that these neighborhoods produce is.

$\tau_r = \{U, \varnothing, (T_5), (T_3, T_4), (T_1, T_5), (T_3, T_4, T_5), (T_1, T_3, T_4, T_5), (T_2, T_3, T_4, T_5)\}$.

The family of r-inverse simply open sets of U is $\text{ISM}_r O(U) = P(U)$.

7 Methods

We apply our strategies to this data in Table 3. Our aim is to classify the sets and measure their exactness and roughness. we defined different approximations based on topological structures. This leaves us to get to the mechanism for decreasing the boundary regions and making it small as possible which is highly important in the rough set context of reducing ambiguity in data and achieving a higher accuracy measure.

8 Results

We present a comparative analysis of the proposed methods with the earlier methods. The methods of M. E. Abd El-Monsef et al. in Definition 2.3, W. S. Amer et al. in Definition 2.4, the current approach in Definitions 3.1, M.K. El-Bably in Definition 2.9, Definition 5.1, and Definition 5.3 are used to calculate the boundary region and accuracy. From Table 3, it is clear that the best of these methods given by using our techniques· In this instance, the boundary areas are canceled. The outcomes are crucial in removing the imprecision associated with rough sets.

9 Discussion

Therefore, based on the measure values obtained, the boundary regions and the accuracy measure for example the set $\{T_3, T_5\}$ using the proposed approaches and

Table 2 Simulated tree structure pairwise similarity

	T_1	T_2	T_3	T_4	T_5
T_1	1	0.33	0.27	0.54	0.83
T_2	0.30	1	0.76	0.75	0.71
T_3	0.22	0.35	1	0.76	0.44
T_4	0.31	0.42	0.71	1	0.20
T_5	0.75	0.33	0.54	0.40	1

Table 3 Application results between the boundary regions and accuracy measures for j ∈ r

A	M. E. Abd El-Monsef		Amer WS		The current method		M.K. El-Bably		Ism-generalization		M-generalization	
	B_r	σ_r	B_r	σ_r	$B_r^{b^*}$	$\sigma_r^{b^*}$	B_r^{sm}	σ_r^{sm}	B_r^{Ism}	σ_r^{Ism}	B_j^{M}	σ_j^{M}
$\{T_1\}$	$\{T_1\}$	0	$\{T_1\}$	0	$\{T_1\}$	0	∅	1	∅	1	∅	1
$\{T_2\}$	$\{T_2\}$	0	$\{T_2\}$	0	$\{T_2\}$	0	∅	1	∅	1	∅	1
$\{T_3\}$	$\{T_2, T_3, T_4\}$	0	$\{T_3\}$	0	∅	1	$\{T_3, T_4\}$	0	∅	1	∅	1
$\{T_4\}$	$\{T_2, T_3, T_4\}$	0	$\{T_4\}$	0	∅	1	$\{T_3, T_4\}$	0	∅	1	∅	1
$\{T_5\}$	$\{T_1, T_2\}$	0.33	$\{T_1\}$	0.50	$\{T_1\}$	0.50	∅	1	$\{T_5\}$	0	∅	1
$\{T_1, T_2\}$	$\{T_1, T_2\}$	0	$\{T_1, T_2\}$	0	$\{T_1, T_2\}$	0	∅	1	∅	1	∅	1
$\{T_1, T_3\}$	$\{T_1, T_2, T_3, T_4\}$	0	$\{T_1, T_3\}$	0	$\{T_1\}$	0.50	$\{T_3, T_4\}$	0.33	∅	1	∅	1
$\{T_2, T_3\}$	$\{T_2, T_3, T_4\}$	0	$\{T_2, T_3\}$	0	∅	1	$\{T_3, T_4\}$	0.33	∅	1	∅	1
$\{T_1, T_4\}$	$\{T_1, T_2, T_3, T_4\}$	0	$\{T_1, T_4\}$	0	$\{T_1\}$	0.50	$\{T_3, T_4\}$	0.33	∅	1	∅	1
$\{T_2, T_4\}$	$\{T_2, T_3, T_4\}$	0	$\{T_2, T_4\}$	0	∅	1	$\{T_3, T_4\}$	0.33	∅	1	∅	1
$\{T_3, T_4\}$	$\{T_2\}$	0.67	∅	1	∅	1	∅	1	$\{T_5\}$	0.67	∅	1
$\{T_1, T_5\}$	$\{T_2\}$	0.67	∅	1	∅	1	∅	1	$\{T_5\}$	0.50	∅	1
$\{T_2, T_5\}$	$\{T_1, T_2\}$	0.33	$\{T_1\}$	0.67	$\{T_1\}$	0.67	∅	1	$\{T_5\}$	0.50	∅	1
$\{T_3, T_5\}$	$\{T_1, T_2, T_3, T_4\}$	0.20	$\{T_1, T_2, T_4\}$	0.40	$\{T_1\}$	0.67	$\{T_3, T_4\}$	0.33	∅	1	∅	1
$\{T_4, T_5\}$	$\{T_1, T_2, T_3, T_4\}$	0.20	$\{T_1, T_2, T_3\}$	0.40	$\{T_1\}$	0.67	$\{T_3, T_4\}$	0.33	∅	1	∅	1
$\{T_1, T_2, T_3\}$	$\{T_1, T_2, T_3, T_4\}$	0	$\{T_1, T_2, T_3\}$	0	$\{T_1\}$	0.67	$\{T_3, T_4\}$	0.50	∅	1	∅	1
$\{T_1, T_2, T_4\}$	$\{T_1, T_2, T_3, T_4\}$	0	$\{T_1, T_2, T_4\}$	0	$\{T_1\}$	0.67	$\{T_3, T_4\}$	0.50	∅	1	∅	1
$\{T_1, T_3, T_4\}$	$\{T_1, T_2\}$	0.50	$\{T_1\}$	0.67	$\{T_1\}$	0.67	∅	1	$\{T_5\}$	0.75	∅	1
$\{T_2, T_3, T_4\}$	$T_2\}$	0.67	∅	1	∅	1	∅	1	$\{T_5\}$	0.75	∅	1
$\{T_1, T_2, T_5\}$	$\{T_2\}$	0.67	∅	1	∅	1	∅	1	$\{T_5\}$	0.67	∅	1
$\{T_1, T_3, T_5\}$	$\{T_2, T_3, T_4\}$	0.40	$\{T_2, T_4\}$	0.60	∅	1	$\{T_3, T_4\}$	0.50	∅	1	∅	1
$\{T_2, T_3, T_5\}$	$\{T_1, T_2, T_3, T_4\}$	0.20	$\{T_1, T_4\}$	0.60	$\{T_1\}$	0.75	$\{T_3, T_4\}$	0.50	∅	1	∅	1
$\{T_1, T_4, T_5\}$	$\{T_2, T_3, T_4\}$	0.40	$\{T_2, T_3\}$	0.60	∅	1	$\{T_3, T_4\}$	0.50	∅	1	∅	1
$\{T_2, T_4, T_5\}$	$\{T_1, T_2, T_3, T_4\}$	0.20	$\{T_1, T_3\}$	0.60	$\{T_1\}$	0.75	$\{T_3, T_4\}$	0.50	∅	1	∅	1
$\{T_3, T_4, T_5\}$	$\{T_1, T_2\}$	0.60	$\{T_1, T_2\}$	0.60	$\{T_1, T_2\}$	0.60	∅	1	∅	1	∅	1
$\{T_1, T_2, T_3, T_4\}$	$\{T_1, T_2\}$	0.50	$\{T_1\}$	0.75	$\{T_1\}$	0.75	∅	1	$\{T_5\}$	0.80	∅	1
$\{T_1, T_2, T_3, T_5\}$	$\{T_2, T_3, T_4\}$	0.40	$\{T_4\}$	0.80	∅	1	$\{T_3, T_4\}$	0.60	∅	1	∅	1
$\{T_1, T_2, T_4, T_5\}$	$\{T_2, T_3, T_4\}$	0.40	$\{T_3\}$	0.80	∅	1	$\{T_3, T_4\}$	0.60	∅	1	∅	1
$\{T_1, T_3, T_4, T_5\}$	$\{T_2\}$	0.80	$\{T_2\}$	0.80	$\{T_2\}$	0.80	∅	1	∅	1	∅	1
$\{T_2, T_3, T_4, T_5\}$	$\{T_1\}$	0.80	$\{T_1\}$	0.80	$\{T_1\}$	0.80	∅	1	∅	1	∅	1
U	∅	1	∅	1	∅	1	∅	1	∅	1	∅	1

the previous approaches such as M. E. Abd El-Monsef in Definition 2.3, Amer WS in Definition 2.4, M.K. El-Bably approach in Definition 2.9, the Currrent approach (b_j*-open sets) in Definition 3.1, the first method in the Definition 5.1.2 and the second method in Definition 5.2.2 respectively.

- M. E. Abd El-Monsef:

The boundary region and the accuracy for the set $\{T_3, T_5\}$.

$B_r(\{T_3, T_5\}) = \{T_1, T_2, T_3, T_4\}$, $\sigma_r(\{T_3, T_5\}) = 0.20$ and accordingly $\{T_3, T_5\}$ is rough set.

- Amer WS:

The boundary region and the accuracy for the set $\{T_3, T_5\}$.

$B_r(\{T_3, T_5\}) = \{T_1, T_2, T_4\}$, $\sigma_r(\{T_3, T_5\}) = 0.40$ and accordingly $\{T_3, T_5\}$ is rough set.

- M.K. El-Bably:

The boundary region and the accuracy for the set $\{T_3, T_5\}$.

$B_r^{sm}(\{T_3, T_5\}) = \{T_3, T_4\}$, $\sigma_r^{sm}(\{T_3, T_5\}) = 0.33$ and accordingly $\{T_3, T_5\}$ is rough set.

- The Currrent approach using b_j*-open sets:

The boundary region and the accuracy for the set $\{T_3, T_5\}$.

$B_r^{b^*}(\{T_3, T_5\}) = \{T_1\}$, $\sigma_r^{b^*}(\{T_3, T_5\}) = 0.67$ and accordingly $\{T_3, T_5\}$ is rough set.

- Ism-Generalization method using $j-$inverse simply open sets:

The boundary region and the accuracy for the set $\{T_3, T_5\}$.

$B_r^{Ism}(\{T_3, T_5\}) = \varnothing$, $\sigma_r^{Ism}(\{T_3, T_5\}) = 1$ and accordingly $\{T_3, T_5\}$is exact set.

- M-Generalization method using j-inverse simply open sets:

The boundary region and the accuracy for the set $\{T_3, T_5\}$.

$B_j^M(\{T_3, T_5\}) = \varnothing$, $\sigma_j^M(\{T_3, T_5\}) = 1$ and accordingly $\{T_3, T_5\}$is exact set.

We can conclude that the proposed approximations were more precise than earlier techniques and helped eliminate uncertainty in real-world scenarios that required precise decisions.

10 Conclusions and future works

The exploration of near open sets holds substantial significance in the broader context of generalized rough set theory. This concept, referred to as near open sets, furnishes researchers with a powerful tool to expand the boundaries of rough sets and enhance the precision of measurements. The original rough set theory faced limitations due to its reliance on equivalence relations, on straining its applicability. To overcome this obstacle, diverse variations of near open sets were introduced, enabling the treatment of imperfect or uncertain knowledge. A central focal point in rough set theory revolves around the reduction of boundary regions, with the ultimate aim of augmenting the precision of decision-making processes. This paper introduces innovative methodologies for extending the scope of rough set theory. Specifically, it introduces a novel concept termed "j-inverse simply open sets" within the context of the j-neighborhood space. These methods facilitate a more robust approximation of sets while alleviating the inherent imprecision found in rough sets. The paper rigorously examines the properties inherent to these methods, which are constructed from a binary relation that in turn gives rise to essential topological structures crucial for the proposed approximation spaces. Surprisingly, the proposed approximation spaces preserve all of the properties of Pawlak's rough sets without the need for extra requirements. This represents a convergence of broad topology and rough set theory. In the future, we hope to investigate more applications of topological notions in rough set.

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