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REVIEW

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Stochastic multi-criteria decision-making: an overview to methods and applications

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Abstract

Background: The alternatives selection problem with multi-criteria in stochastic form variables is called as stochastic multi-criteria decision-making. The stochasticity of the criteria is considered using stochastic dominance, prospect theory, and regret theory.

Main text: In this paper, a total 61 papers are reviewed and analyzed based on method(s) used in stochastic multi-criteria decision-making problem, method used in stochasticity, specific objective, application area, and so on classification. All papers with respect to classification aspects are examined their real or empirical applications. Moreover, the studies are statistically investigated to present the latest trends of stochastic multi-criteria decision-making.

Conclusions: This detailed review study ensures a comprehension for researchers on stochastic multi-criteria decision-making in respect of showing up-to-date literature and potential research areas to be concentrated in the future. It is observed that the stochastic multi-criteria decision-making problem has an attractive approach by researchers.

Keywords: Stochastic decision-making, Probability, Stochastic dominance, Regret theory, Prospect theory

1 Background

MCDM is a research area of management science and operations research which has been extensively analyzed by researchers [4, 5, 31]. It is related to assessing, selecting, and evaluating options from the best to the worst in regard to conflict criteria using expert(s) preferences [1]. The SMCDM aims to select from several criteria, mathematically expressed as neither real nor fuzzy numbers or random variables [50]. While SMCDM computes all kinds of ways to achieve a duty, fuzzy MCDM tries to find one best way to do the duty [7]. There are two review papers on SMCDM by Tervonen and Figueira [52] and Antucheviciene et al. [2]. Tervonen and Figueira [52] presented a detailed literature review for methods and describe a unified stochastic multi-criteria acceptability analysis methods (SMAA). SMAA application is listed with the definition of particularities of each one to introduce historical comprehension into the practices included in the methodology practice. They also remark the highlights in the methodology for future directions. Antucheviciene et al. [2] presented fuzzy and stochastic MCDM methods for solving civil engineering problems. Unfortunately, there is no detailed review of SMCDM approaches. However, there have been several SMCDM approaches (Table 1). Hence, we review the literature about SMCDM approaches using academic databases. On the other hand, SMCDM approaches should receive greater attention in later studies [33].

Literature-related SMCDM, which a total of 61 papers, were analyzed ranged from 1996 to December 2018. The main contributions of our paper are summarized as follows: (1) it determines the SMCDM approaches that have been combined with stochastic parameters, (2) it represents method(s) used in SMCDM problem: AHP, TOPSIS, PROMETHEE, ELECTRE, VIKOR, AHP-TOPSIS hybrid methods, ANP, (3) which stochasticity used in SMCDM problems as stochastic dominance (SD) degree, prospect theory (PT), regret theory (RT), and others that have been further used by SMCDM approaches, (4) it shows the countries of the published papers, and (5) the trend of SMCDM is also determined for future studies.

The rest of the paper is given as follows: a summary overview of the fundamentals of SMCDM is given in Sub-section 1. While Section 2 presents the review methodology, the stochastic MCDM methods and applications are analyzed in Section 3. Results and

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Table 1 Summary of MCDM approaches

Abbreviation	Method	Description
SAHP	Analytic hierarchy process	A hierarchical pairwise comparison considering stochastic variables
SANP	Analytic network process	Evaluation of the dynamic multi-directional relationship between the decision criteria using stochastic variables
STOPSIS	Technique for order of preference by similarity to ideal solution	A MCDM technique based on the concept of choosing the solution with distance from ideal solution considering stochastic variables
SPROMETHEE	Preference ranking organization method for enrichment of evaluations	An outranking method based on a pairwise comparison of alternatives to defined criterion using stochastic variables
SELECTRE	Elimination et choix traduisant la realité	An outranking method based on pairwise comparisons to determine the concordance and discordance sets using stochastic variables
SVIKOR	Visekriterijumska Optimizacija IKompromisno Resenje	Method for determining the compromise ranking-list of a set of alternatives using stochastic variables
SEDAS	The evaluation based on distance from average solution	It is based on distances of each alternative from the average solution with respect to each criterion

discussions are detailed in Section 4. Lastly, limitations, recommendations, and conclusions are presented for future directions in Section 5.

1.1 The fundamentals of SMCDM

In this paper, we first presented the fundamentals of the RT [39, 71], PT [13, 39] and SD ([62–64]; Maciej [37, 50, 69]).

1.2 Regret theory

RT is firstly developed by Bell [6] and Loomes and Sugden [30]. The RT is a novel significant reasoning method and the preferences are not required to be transitive. Regret theory is a nontransitive model describing preferences by a bivariate utility function. The details of the basic concept of the utility function can be analyzed from the article of [6, 8, 30, 39, 68].

1.3 Prospect theory

The PT is firstly proposed by Kahneman and Tversky [22]. The optimal alternative is selected with respect to the prospect values of all alternatives. It is defined by the value and the probability weight function. The outcome is defined as the gain when the existing wealth surpasses the reference point. On the other hand, the outcome is defined as the loss. The PT underlines the difference between expectation and result, rather than the result itself; hence, the selection of reference point is very important [23, 53].

1.4 Stochastic dominance

Two groups for two classes of utility functions classify the rules of SD [61]. While the first group comprises of first, second, and third-degree stochastic dominance, the second group comprises first-degree stochastic dominance, second inverse stochastic dominance, third inverse SD of the first type and third inverse SD of second type. The first group is utilized in the gains domains, but the second group is used in the losses domain [37]. The description of SD rules can be analyzed in Zhang et al. [69].

2 Review methodology

This review study was implemented with articles from journals that ensure significant insights for researchers studying on SMCDM. Hence, a research methodology is given in Fig. 1 is followed in this study.

First, we gather related article from important databases with appropriate search hints (*Stochastic* multi-criteria decision-making OR stochastic multi-attribute decision-making AND AHP; Stochastic multi-criteria decision-making OR Stochastic multi-attribute decision making AND TOPSIS; Stochastic multi-criteria decision-making OR Stochastic multi-attribute decision-making AND VIKOR; Stochastic multi-criteria decision-making OR Stochastic multi-attribute decision-making AND ELECTRE; Stochastic multi-criteria decision-making OR Stochastic multi-attribute decision-making AND PROMETHEE; Stochastic multi-criteria decision-making OR Stochastic multi-attribute decision-making AND dominance degree; Stochastic multi-criteria decision-making OR Stochastic multi-attribute decision-making AND PT; Stochastic multi-criteria decision-making OR Stochastic multi-attribute decision-making AND RT). Hence, a wide search was implemented in the keywords, abstract, and title of scholarly papers. The main library databases, which are Springer, Science Direct, Wiley, Taylor & Francis, Emerald, Hindawi, ASME, MDPI, World Scientific, and IEEE, cover most of the papers are used during the review process. Unpublished working papers and thesis were removed from this study. An Excel sheet was used to examine, classify, and document of the papers with the following dimensions:

- Year: publication year;
- Journal: journal title;

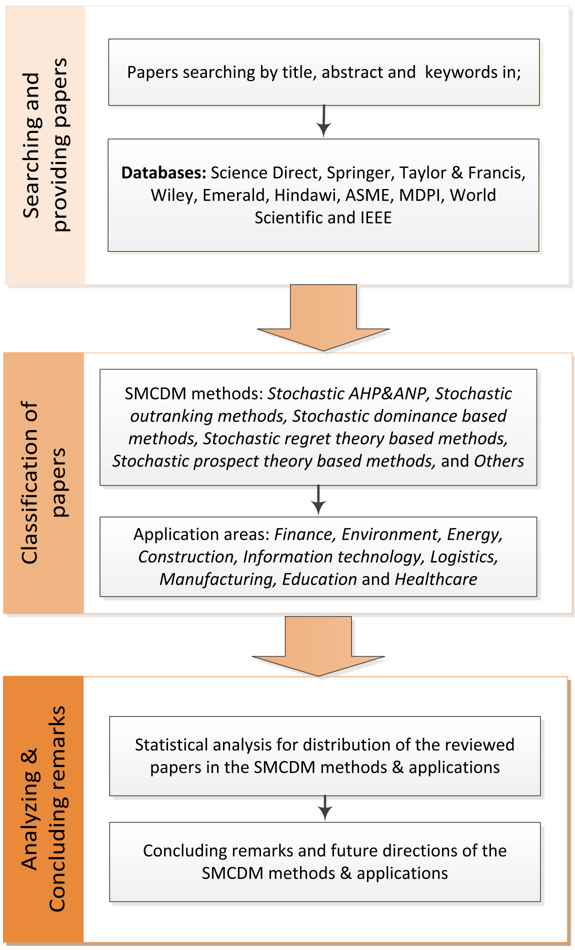

Fig. 1 Research methodology of the SMCDM review

- Country: country where the study was being conducted (In general, country of the first author is considered);
- Method(s) used in SMCDM problem: AHP, TOPSIS, PROMETHEE, ELECTRE, VIKOR, AHP-TOPSIS hybrid methods, ANP;
- Method used in stochasticity: SD degree, PT, RT, and etc.;
- Specific objective: short aim of the study
- Application area: applied areas are construction (C), education (ED), energy (EN), environment (ENV), finance (F), healthcare (H), information technology (IT), logistics (L) and manufacturing (M);
- Statistical distribution type used in SMCDM problem.

Second, a classification is performed according to applied methods used for SMCDM problem. Ultimately, we analyze the studies by considering statistical results the studies distributions and concluding remarks of future directions.

3 Stochastic MCDM methods and applications

In this section, papers are presented according to SMCDM methods as presented in Figs. 2 and 3.

3.1 Stochastic AHP and ANP methods and applications

AHP is based on the hierarchical MCDM problem that comprises attributes, alternatives, and goal. Pairwise comparisons are applied in each hierarchical level with judgments using real values received from the scale of Saaty [47]. In SMCDM knowledge, imprecise preferences of decision-makers must be converted into the stochastic pairwise comparisons [9]. To get crisp values of a stochastic pairwise comparison, the conversion is applied with respect to the probability density functions with related parameters. On the other hand, ANP can be used to model SMCDM problems. It is an appropriate approach for solving decision-making problems with the inclusion of interaction and dependence among criteria and sub-criteria [67]. In SMCDM literature, several papers contributed to both methodologically by proposing stochastic based AHP and its variations and applicably by finding solutions in different areas. The following studies were retrieved in terms of application novelty in SMCDM knowledge using AHP, FAHP, or ANP.

Ramanathan [43] adapted stochastic programming to multiplicative AHP context. The process of weight derivation using multiplicative AHP was considered. Stochastic goal programming is used for developing to derive the maximum likelihood values of weights. Stam and Silva [49] proposed two measures of rank reversal probabilities in the AHP resulting from pairwise judgments. Van den Honert [55] examined the effect of uncertainty in the pairwise judgements or ratings of alternatives as a probability distribution. Cobuloglu and Büyüktahtakın [9] presented SAHP for biomass selection problem. They used the beta distribution and approximating its median. The logarithmic least squares method is applied to measure the consistency. Ubando et al. [54] applied SAHP in algal cultivation systems assessment for sustainable production of biofuel. Zhao and Li [70] proposed a model to assess the performance of strong smart grid based on the SAHP and fuzzy TOPSIS. A sensitivity analysis was also implemented to prove the robustness of the proposed approach as in Ubando et al. [54]. Zhang et al. [67] presented a stochastic multi-criteria assessment developed by applying the SANP-GCE weight calculation approach. The proposed SANP—game cross-evaluation (GCE) handled the uncertainties and inconsistencies of expert opinions. Finally, the use of ArcGIS helped to visualize vulnerabilities and sensitivities spatially, thus making the decision process more intuitive. Moreover, the criteria weights constituting Nash equilibrium points that determined by GCE improved the objectivity of SANP. Rabelo et al. [42] used hybridized

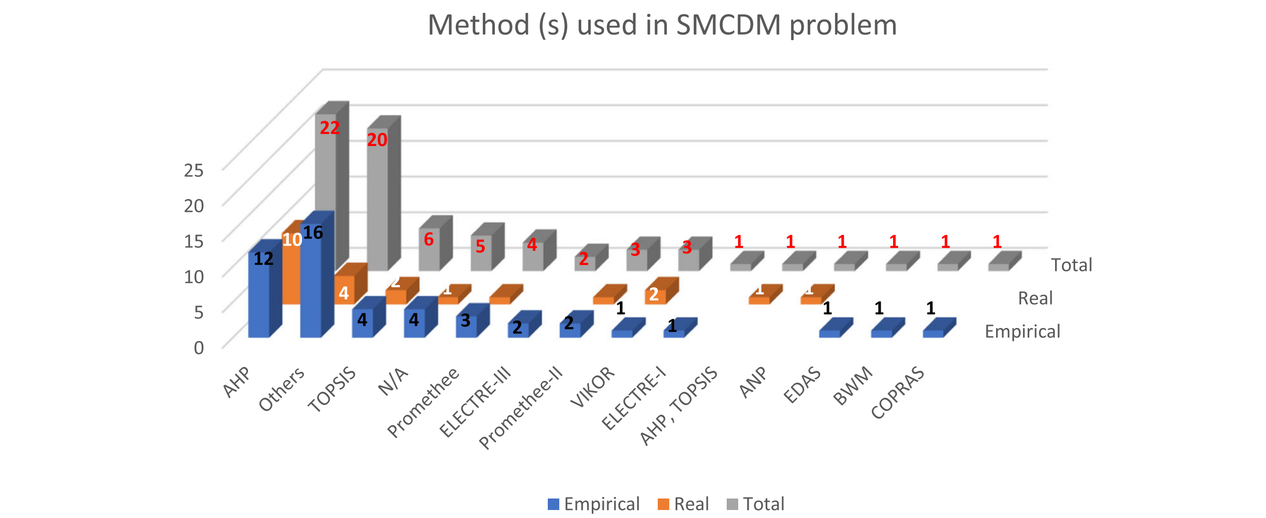

Fig. 2 Method(s) used in SMCDM problem

SD–DES simulation models and AHP for value chain analysis. Banuelas and Antony [3] applied SAHP for selecting the best suitable technology for the domestic appliance platform. Four design concepts and eight criteria were considered.

Kim et al. [25] applied SAHP and knowledge-based experience curve (EC) to rank restoration needs. AHP and SAHP are compared for ordering restoration needs of cultural heritage. Minmin and Li [35] proposed SAHP and fuzzy AHP for credit evaluation. Jing et al. [20, 21] contributed to the SAHP application domains. In the first paper, they incorporated stochastic and fuzzy uncertainty into the traditional AHP as fuzzy SAHP. In the second one, they proposed a hybrid stochastic-interval AHP method to reflect uncertainty by combining lexicographic goal programming, probabilistic distribution, interval judgment, and Monte Carlo simulation.

Apart from application novelties of reviewed SAHP-related papers, some are available in the current knowledge which includes methodological novelties. They are summarized as follows: Phillips-Wren et al. [40] presented SAHP in the context of a real-time threat criticality detection decision support systems. Hahn [15] proposed two stochastic formulations of the AHP using Bayesian categorical data. While the first model used a multinomial logit model, the second one used independent multinomial probit model. Eskandari and Rabelo [11] presented a stochastic approach for calculating the

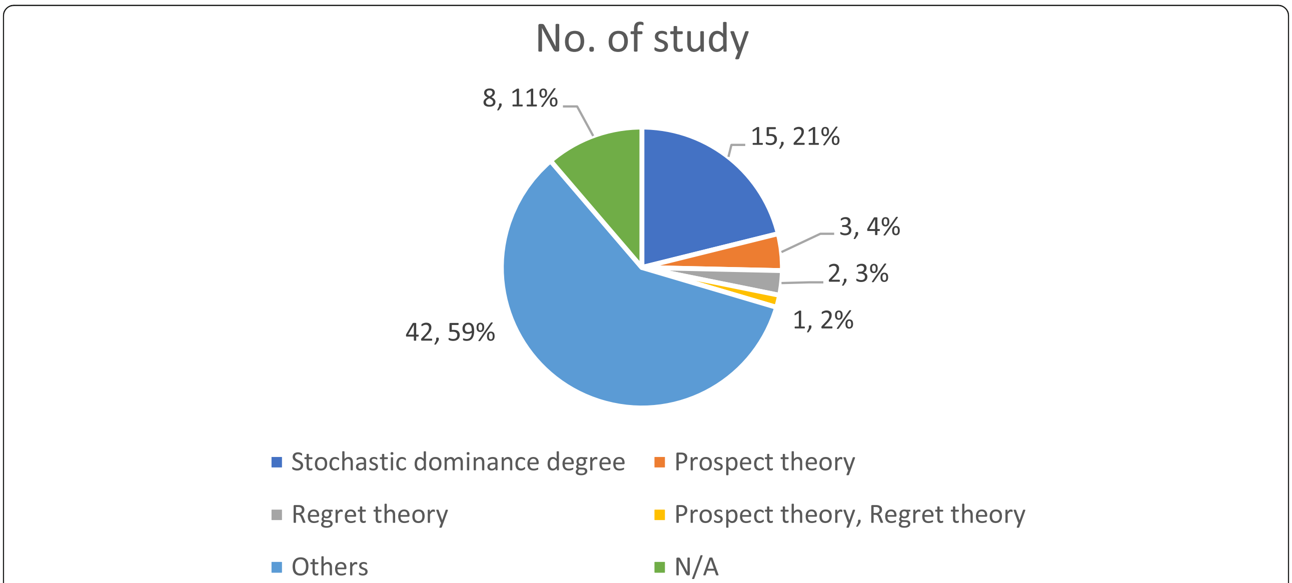

Fig. 3 Method used in stochasticity

variances of the AHP weights using Monte Carlo simulation. Wanitwattanakosol et al. [57] used AHP for input feature selection in logistics management. Ramanujan et al. [44] developed a SAHP approach and implemented it for prioritizing design for environment strategies. Jalao et al. [18] proposed an AHP model changing stochastic preferences of the decision-maker. AHP with stochastic multi-criteria acceptability analysis (SMAA) is combined by Durbach et al. [10]. The consistency of judgements is analyzed using a simulation experiment.

3.2 Stochastic outranking methods and applications

PROMETHEE method was proposed by Brans et al. [32]. Stochastic PROMETHEE (SPROMETHEE) is a solid member of SMCDM methods. The probability distributions are used for the input parameters instead of real values [33]. In this category, we can also mention ELECTRE and its family with various versions. SMCDM method, which is based on the SD degree using the simple additive weighting method, was proposed by Zhang et al. [69]. PROMETHEE-II was proposed to acquire the alternatives ranking result based on SD degree. Hyde and Maier [17] presented a stochastic uncertainty and distance-based analysis in Excel using Visual Basic. While Marinoni [33] proposed SPROMETHEE in GIS, Marinoni [34] compared the results of a stochastic multivariate PCA and the results of stochastic outranking evaluations. Maciej Nowak [37] showed how to employ the concept of the threshold in the stochastic case using stochastic dominance. The concept of pseudo-criteria was used. Zaras [63] suggested an approach using SD for a reduced number of attributes. Rogers and Seager [46] presented a method based on stochastic multi-attribute life cycle impact assessment. Random variables with probability distributions used the consequence of the alternative according to criteria by Liu et al. [28, 29]. At first, the alternative pairwise comparisons dominance degree matrix according to each criterion was implemented with probability distributions comparison. Then, an overall dominance degree matrix was constructed using PROMETHEE II. Zhou et al. [71] proposed a gray SMCDM approach based on a combination of SMAA-ELECTRE, with criteria values that extended gray random variables. With this approach, it contributes a new way to solve SMCDM problems with imprecise, uncertain, and/or missing preference information, and also they determine that gray number is a powerful tool to express uncertainty in MCDM problems. Keshavarz Ghorabaee et al. [24] proposed a stochastic EDAS method using the normal distribution.

3.3 Stochastic dominance-based methods and applications

SD aims to choose the best alternative that dominates another. Some papers on SD-based methods have been proposed. Nowak [38] combined SD and interactive approach to suggest a new procedure for a discrete SMCDM problem. Nowak [37] aimed to present how to use the concept of the threshold in the stochastic case. Unlike mean-risk analysis, SD can be implemented into models of preferences versus risks. Zaras [63] recommended the multi-criteria SD to reduce attributes number. Zaras [64] made the standardization by the dominance notion extension to evaluate all types (fuzzy or probabilistic, deterministic). Deterministic, stochastic, or fuzzy are examined as three kinds of evaluations that are defined as mixed-data dominances. Zaras [62] proposed a rough sets methodology for the preferential information analysis. Xiong and Qi [59] applied interval estimation for converting SMCDM to IMCDM using TOPSIS. Zhang et al. [69] used a simple additive weighting method in SD degree matrix for PROMETHEE-II. Mousavi et al. [36] presented a fuzzy-stochastic VIKOR approach. Triangular fuzzy numbers and associated linguistic variables were used in MCDM problem. The performance distribution is generated by applying Monte Carlo simulation. Lastly, VIKOR was implemented to assess probability distributions for each alternative on each criterion. Jiang et al. [19] used SD rules in the classical TOPSIS method. The probability distributions for both stochastic and discrete variables are defined and determined. Tavana et al. [51] extended the VIKOR method and improve a methodology to solve problems of MCDM with stochastic data. They presented a case study to evaluate 22 bank branches performance efficiency using SVIKOR. Zhao and Li [70] proposed fuzzy TOPSIS and stochastic AHP to evaluate the strong smart grid performance. While fuzzy TOPSIS method is applied to evaluate the performance of the smart grid, stochastic AHP method is used to get the sub-criteria weights. Yang and Huang [60] presented a dynamic stochastic decision-making method. Firstly, the proposed approach obtained time-sequence weights by combining time-degree theory and TOPSIS. Attribute weights were determined based on the characteristics of normally distributed vertical projection distance and stochastic variable variances. Decision-making information is then integrated from time-sequence weights and the attribute via related operators, to obtain the stochastic normally distributed comprehensive decision-making matrix constituted by target single dimensions. Finally, the priority sequence of alternative solutions was provided using order relation criteria. Kolios et al. [26] proposed stochastic TOPSIS in selecting offshore wind turbines support structures. A TOPSIS-based method considering stochastic inputs (statistical distributions) was proposed for an offshore wind turbine supports the structure selection process. Based on the collected data, a sensitivity analysis was illustrated the required number of simulations for the required accuracy and performed an assessment of the results based on weighting of the respondents' perceived expertise. Liang et al. [27] presented a new method based on disappointment SD with respect to the SMCDM problem with criterion 2-tuple aspirations. The overall disappointment SD each alternative degree over the aspiration alternative is calculated to determine the ranking result. Wu et al. [58] proposed an interval number explanation with the distribution of probability.

3.4 Stochastic regret theory-based methods and applications

RT is a novel significant reasoning method that does not involve preferences to be transitive. It is a nontransitive model to show preferences by a bivariate utility function, which takes the feelings of regret and rejoice into consideration [39]. The number of RT-based methods is scarce and the number of paper should be increased. Zhou et al. [71] proposed a gray stochastic MCDM approach based on TOPSIS and RT. Discrete and continuous gray numbers were proposed to represent the values of criteria. At first, RT was applied to get the utility and regret value concerning the criteria. Then, the TOPSIS method was applied to rank the alternatives with respect to the overall perceived utility intervals. Two algorithms are proposed which take decision-makers prospect preference and regret aversion by Peng and Yang [39]. The score function based on regret and PT is proposed for two new interval-valued fuzzy soft approaches. A novel interval-valued fuzzy distance measure axiomatic definition is constructed.

3.5 Stochastic prospect theory-based methods and applications

PT assumes that the decision-maker(s) will opt for the optimum alternative with respect to all alternative prospect value. It is decided with probability weight function and the value. Peng and Yang [39] used PT to calculate score function. Liu et al. [28, 29] developed a MCDM based on PT. It is compared with classical MCDM methods. The result of the proposed method based on PT is compared with expected utility theory. Tan et al. [50] aimed to develop a new method based on combining PT with stochastic dominance. The proposed approach is compared with other SMCDM methods based on stochastic dominance. Hu and Yang [16] proposed a dynamic SMCDM based on cumulative PT and set pair analysis. Zhou et al. (2017) proposed a gray SMCDM approach based on distance measures and PT that is integrated with discrete gray numbers. The proposed approach is TODIM that aims to select the best alternative. Gao and Liu [13] proposed an approach to solving the interval-valued intuitionistic fuzzy SMCDM problem. A new precision score function was suggested based on the hesitation, nonmembership, and membership degrees to transform the interval-valued intuitionistic fuzzy number into a computational numerical value. A new criteria weighting model was put forward based on the least square method, the maximizing deviation method, and PT.

3.6 Others

Some papers are not compatible with subtitle as RT, SD degree, and etc. Zarghami et al. [66] presented fuzzy-stochastic MCDM approach by combining the stochastic and fuzzy sets for OWA operator. Random variables with probability mass functions or known probability density functions in SMCDM approach were used by Fan et al. [12]. They applied pairwise comparison for evaluating alternatives with a random variable. They used identification rule, superior, indifferent, and inferior probabilities on pairwise comparison. Ren et al. [45] proposed a SMCDM approach using differences between the superiorities and the inferiorities. Zarghami and Szidarovszky [65] presented a new approach fuzzy-stochastic-revised ordered weighted averaging. The stochastic and fuzzy sets are combined in a revised OWA operator. Zarghami and Szidarovszky [65] proposed stochastic fuzzy ordered weighted averaging approach. Simulation model and fuzzy linguistic quantifiers are applied to the inputs of the approach and obtaining the optimism degree of the decision-maker(s), respectively. Prato [41] considered probability distributions and the other information required to implement the method for SMCDM method. The method can be applied to order any set of management actions for which the stochastic attributes of outcomes can be is willingly suitable. Wang et al. [56] proposed gray SMCDM problems with incompletely uncertain criteria weights. An optimal programming model based on the sorting vector closeness degree is constructed. It is solved using a genetic algorithm to get optimum criteria weights when the criteria weights were uncertain.

4 Results and discussions

4.1 Classification of papers

A total of 61 papers on SMCDM approaches were analyzed in this literature review. The majority of the 57(94%) belong to journal articles, a number of 3(5%) are presented at selected congress proceedings, and very few 1(2%) are published as a book chapter.

Then, the data are also used to model the evolution of SMCDM approaches in time, by fitting the distribution of the number of studies during the period of 1996–2017 through a regression analysis. It is analyzed with a confidence level of 95%. By this means, the data compiled are fitted to polynomial regression models separately, as shown in Fig. 4.

From Fig. 4, it can be simply recognized that after 2012, there is a vital increase in the publishing of papers. Furthermore, the literature review is classified by country of origin for each study, resulting in the 9 portions and represented in the pie graph (Fig. 5). China accounts

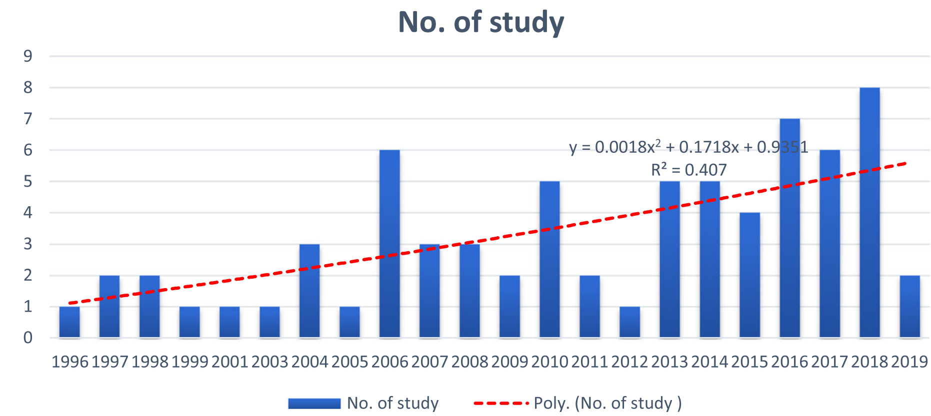

Fig. 4 Number of papers with respect to the total number of studies

for almost 22 (36%) of all papers relevant to the SMCDM approaches. USA, Canada, Iran, and Poland are also prolific in the use of SMCDM (18%, 8%, 7%, and 5% respectively). The rest of the countries have a rather testimonial presence (Germany, South Africa, UK, South Africa, Australia, Finland, Georgia, Hungary, Lithuania, Netherlands, Philippines, Republic of Korea, and Thailand) with 2(3%), 2(3%), 2(3%), 1(2%), 1(2%), 1(2%), 1(2%), 1(2%), 1(2%), 1(2%), 1(2%), 1(2%), and 1(2%), respectively.

Related to the area of application, "finance" take up more than a quarter of the application in SMCDM. Thirty-three percent of total papers ($n = 20$) are focused in this application area (Fig. 6). They concentrate on particular problems such as investment project selection, computer development project selection, luxury automobile selection, credit evaluation, enterprise selection, and bank investment evaluation. Another most studied application area is "environment" by 18% of total papers ($n = 11$). "Energy," "construction," "information technology," and "logistics" are probably in the most delicate disciplines. Other areas of application such as "manufacturing," "education," and "healthcare" are also seldom selected by the authors in terms of SMCDM. Empirical studies are presented by 21% of total papers ($n = 13$) without presenting on a real-world application. Thus, we count them in group N/A.

It is clearly seen from Fig. 7 that among the proposed methods used in stochasticity, SD degree is deemed as the second most applied method after the "others" group

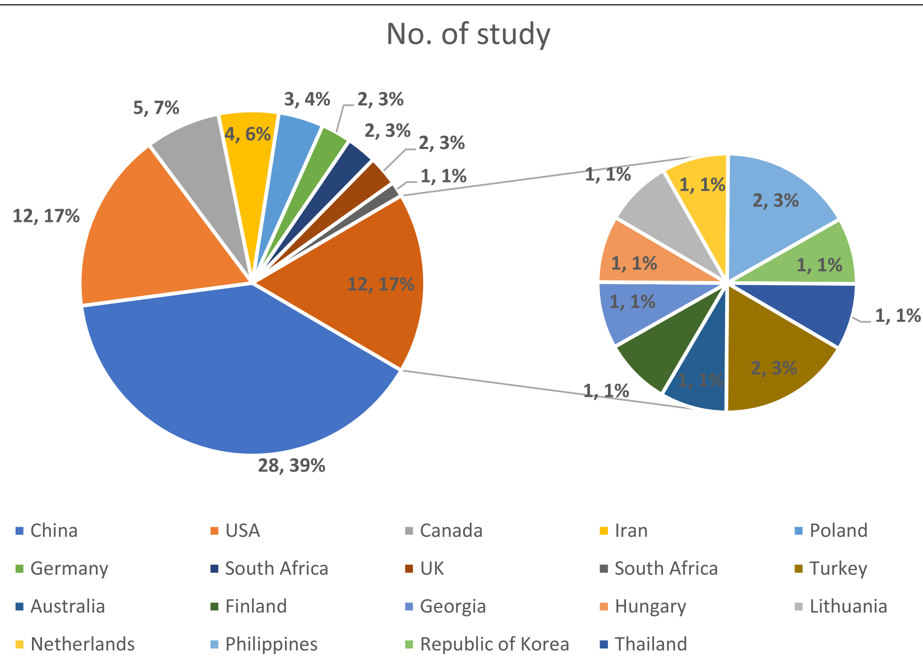

Fig. 5 Distribution of papers by country of origin

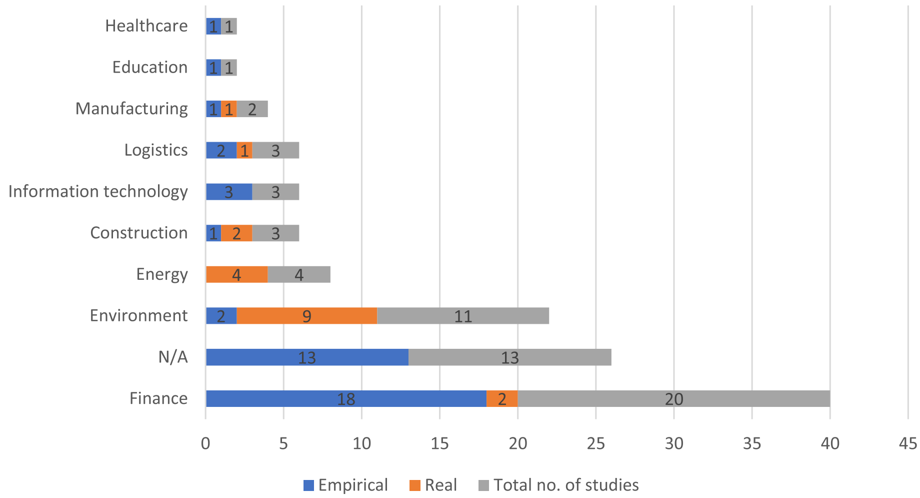

Fig. 6 Distribution of papers by application area and application type

that includes interval estimation, nonlinear programming, probability theory, Bayesian categorical data, stochastic data envelopment analysis, fuzzy AHP, expected probability degree, gray stochastic variable, weighted arithmetic averaging operators, alternative similarity scale, and genetic algorithm. This group is implemented to most of the application areas in the sense of this review excluding information technology.

The top four journals for the number of published papers are presented in Fig. 8. European Journal of Operations Research has the most publications on SMCDM (11; 18%), followed by Mathematical Problems in Engineering (4; 7%), Computers and Industrial Engineering (3; 5%), and Knowledge-Based Systems (3; 4%). Of the journals, Decision Sciences, Information Sciences, International Transactions in Operational Research and Journal of Multi-Criteria Decision Analysis have 2 papers (3% each). Other journals or book chapters contain 1 entry (2% each).

Different statistical probability distributions are used in papers as uniform, normal, Weibull, exponential, binomial, triangular, beta, discrete, lognormal, loglogistic, and gamma that is presented in Table 2. The effects on SMCDM should be analyzed in detail.

4.2 Discussion and future remarks

In literature, the SD degree proposed in the literature is mostly based on the first-degree SD rule. Hence, the higher-order SD degrees for different risk preference styles are also interesting for further studies. In literature, the researcher mostly presented empirical studies rather than a real case study. Hence, a more real case study should be presented for analyzing the proposed SMCDM

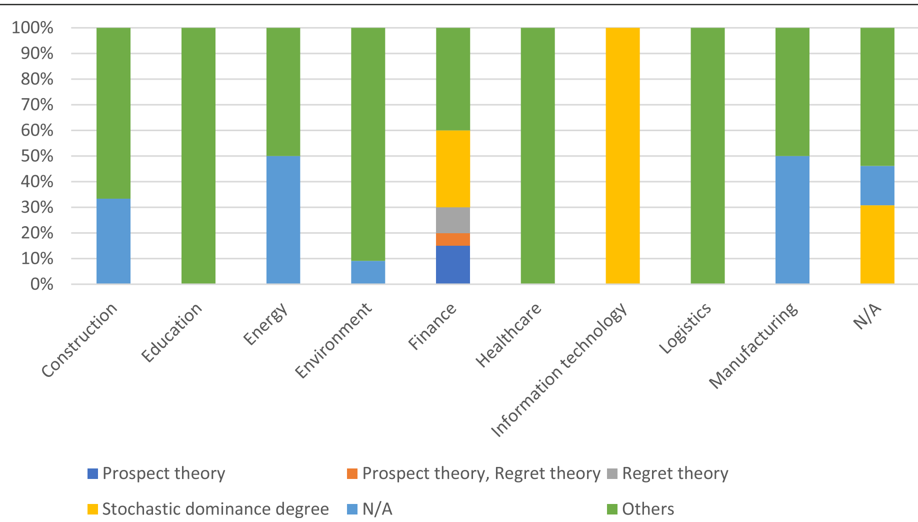

Fig. 7 Distribution of papers in terms of the application area and proposed method used in stochasticity

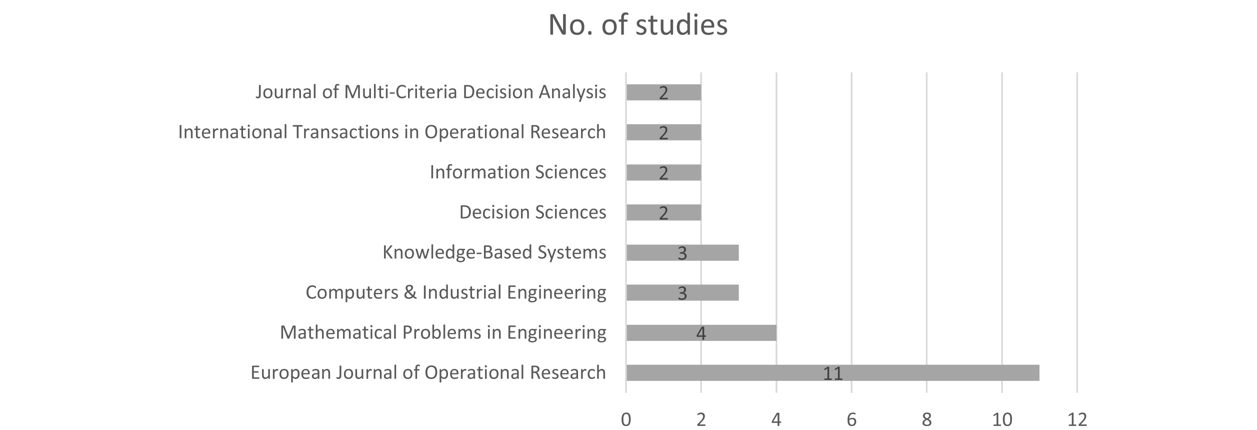

Fig. 8 Distribution of papers in terms of the top four journal source titles

approaches. Developing a decision support system and open-access source for the proposed approaches are suggested to analyze and improve the SMCDM. Interval-valued intuitionistic fuzzy set, interval-valued fuzzy soft sets, the trapezoidal fuzzy number, Triangular fuzzy numbers are combined with stochastic MCDM approaches. Interval type-2 fuzzy sets, Pythagorean fuzzy sets, hesitant fuzzy sets, neutrosophic fuzzy sets should be combined with stochastic MCDM approaches. The number of RT-based methods is scarce and the number of paper should be increased.

The importance of the weight for the criteria can be calculated using AHP, ANP, best-worst method, SWARA, SAW, and DEMATEL approaches. While some extension of stochastic AHP and ANP is applied in literature, the extension of the best-worst method, SWARA, SAW, and DEMATEL based on stochasticity should be developed for future studies. On the other hand, the rankings of the alternatives are calculated proposing TOPSIS, VIKOR, PROMETHEE, and ELECTRE using stochasticity as RT, SD, and PT. For further studies, TODIM, COPRAS, GRA, Qualiflex, information axiom, and Choquet integral should be developed. As a conclusion, SMCDM approaches should receive greater attention in the future since they offer better insight into multi-criteria evaluation results [33].

Table 2 Statistical probability distributions used in SMCDM studies

Distributions used in SMCDM problem	Reference
Uniform	Xiong and Qi [59]; Zhou et al. (2016); Minmin and Li [35]; Jing et al. [21]; Hyde and Maier [17]; Marinoni [34]; Cobuloglu and Büyüktahtakın [9]; Zhao and Li [70]; Marinoni [33]; Zhou et al. [71]
Normal	Xiong and Qi [59]; Ramanathan [43]; Peng and Yang [39]; Tavana et al. [51]; Eskandari and Rabelo [11]; Kim et al. [25]; Szidarovszky and Szidarovszky (2009); Marinoni [34]; Zhang et al. [67]; Yang and Huang [60]; Zhou et al. [71]; Kolios et al. [26]; Shengbao and Chaoyuan [48]; Keshavarz Ghorabaee et al. [24]
Weibull	Hyde and Maier [17];
Exponential	Van den Honert [55];
Binomial	Phillips-Wren et al. [40]; Hahn [15]; Hu and Yang [16];
Triangular	Banuelas and Antony [3]; Zarghami and Szidarovszky [65]; Marinoni [34]; Prato [41]; Cobuloglu and Büyüktahtakın [9]; Zhao and Li [70]; Marinoni [33]; Marinoni [33]
Beta	Jing et al. [20]; Jalao et al. [18]; Marinoni [34]; Cobuloglu and Büyüktahtakın [9]; Zhao and Li [70]; Marinoni [33]
Discrete	Stam and Silva [49]; Tan et al. [50]; Zaras [64]; Zaras [62]; Maciej Nowak [37]; Wang et al. [56]; Zhou et al. [71]; Zhou et al. [72]; Zaras [63]
Lognormal	Hyde and Maier [17]; Marinoni [34];
Loglogistic	Hyde and Maier [17];
Gamma	Marinoni [34];
Others (3-parameter Weibull, Smallest extreme value, Chi-Square, Logbeta, Posterier, Multinomial, PERT, InvGauss, Pearson 5, Gaussian, Dirac's delta function)	Mousavi et al. [36]; Ramanathan [43]; Stam and Silva [49]; Hahn [14]; Hahn [15]; Jing et al. [20]; Ramanujan et al. [44]; Hyde and Maier [17]; Durbach et al. [10];

5 Conclusion

In this paper, we presented a comprehensive review on SMCDM applications and approaches. SMCDM have increased popularity in MCDM problems in an extensive range of applications and approaches because of its ability to implement higher degrees of ambiguity and uncertainty in recent years. We contribute several standpoints to the literature as follows: (1) SMCDM approaches are determined that have been integrated with stochastic parameters, (2) it represents method(s) used in SMCDM problem: AHP, TOPSIS, PROMETHEE, ELECTRE, VIKOR, AHP-TOPSIS hybrid methods, ANP, (3) which stochasticity used in SMCDM problems as SD degree, PT, RT, and others that have been further used by SMCDM approaches, (4) the countries of the author(s) related published papers are presented, and (5) the trend of SMCDM is determined how it will continue in the future. We observe and expect that the number of SMCDM approaches and applications will increase because of the complexity and advanced degrees of vagueness, ambiguity, and uncertainty in MCDM problems.

Abbreviations

MCDM: Multi-criteria decision-making; SAHP: Stochastic analytic hierarchy process; SANP: Stochastic analytic network process; SEDAS: Stochastic the evaluation based on distance from average solution; SELECTRE: Stochastic elimination et choix traduisant la realité; SMCDM: Stochastic multi-criteria decision-making; SPROMETHEE: Stochastic preference ranking organization method for enrichment of evaluations; STOPSIS: Stochastic technique for order of preference by similarity to ideal solution; SVIKOR: Stochastic Visekriterijumska Optimizacija I Kompromisno Resenje

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